Customized Offers in Airline Revenue Management

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Abstract

Improvements in information technology could soon allow airlines to generate customized fare offers for individual passengers. By strategically targeting offers towards the right customers at the right time, airlines could increase revenues by stimulating additional bookings from price-sensitive travelers while encouraging more price-inelastic travelers to buy-up to higher price points. In this work, we introduce a "dynamic availability" method for generating customized fare offers through an assortment optimization and dynamic pricing framework.

We propose a general model for decoupling the customized offer generation problem from the well-studied airline revenue management problem. After generating a baseline assortment and observing a passenger's characteristics, an airline can choose to customize that passenger's offer by either adjusting the products in the assortment or changing the offered prices for those products. Unlike other recent approaches for personalized assortment optimization, our method applies customization after traditional RM optimization, making the heuristics compatible with current airline RM methods and systems.

For implementation, we propose a straightforward heuristic approach based on simple estimates of passenger willingness-to-pay. The heuristics are simulated in the Passenger Origin-Destination Simulator (PODS), a complex airline revenue management simulation environment that takes into account passenger choice and competition. The results show that the heuristics can be revenue positive for airlines and stable in competitive environments.

Keywords: airline revenue management, personalization, assortment optimization, dynamic pricing, New Distribution Capability

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1. Introduction

Since the development of the first sophisticated airline revenue management systems nearly three decades ago, airlines have managed their seat inventory relatively blind to the characteristics of

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the customer making a request. While airlines attempt to segment customers into high willingness-to-pay and low willingness-to-pay groups using techniques like advanced purchase rules, restricted fare structures, and point-of-sale control, observed customer characteristics have generally not been modeled explictly into an airline revenue management framework.

This limitation is largely a relic of technology—through traditional distribution systems, airlines have limited opportunities to identify customer characteristics or present customized offers to individual passengers. However, two technological advancements have the potential to give airlines additional flexibility in customizing their offers. The first is the rise of direct channel (web) bookings, where customers searching and buying on an airline's website may be logged in to their frequent flier account, providing the airline with a rich history of past travel purchases. The second is the introduction of the New Distribution Capability (NDC) by the International Air Transport Association. Among other features, NDC enables the creation of "personalized offers," in which customers could be offered a unique bundle of fare products and add-ons with an offer-specific price (Hoyles, 2015). This "dynamic" offer generation capability has caused some practitioners to speculate that the airline industry is heading for a future without traditional booking classes, in which prices are generated dynamically "on-the-fly" (Westermann, 2013; Bala, 2014).

Despite these advancements, airline revenue management models do not yet account for customer characteristics when computing booking limits or bid prices, and the academic literature has only very recently started to theoretically tackle the issue of "customized" revenue management. The few papers that have started the discussion (Golrezaei et al., 2014; Bernstein et al., 2015; Chen et al., 2015b; Besbes and Sauré, 2016; Gallego et al., 2016) have proposed models and related heuristics for customized assortment optimization or dynamic pricing, but these models are largely not designed for the specific conditions of the airline revenue management problem nor the current state of the practice in airline RM.

The primary focus of this paper is to introduce a framework for formulating the customized airline revenue management problem when customer characteristics (such as purpose of travel or an estimate of willingness-to-pay or price elasticity) can be observed, inferred, or guessed about invididual passengers arriving to book. The general model relies on both an assortment optimization and a dynamic pricing framework, and works by suggesting a heuristic adjustment to either the assortment of fare products or the vector of fares that would be ordinarily offered by a traditional airline RM system. The model therefore is highly compatible with existing leg-based or network airline revenue management systems. Given real-world constraints faced by airlines, the focus of the analysis in this paper is less on finding "optimal" solutions that assume perfect knowledge of the consumer choice function, and more on demonstrating that easier-to-implement heuristics can increase airline revenues in a fashion that is stable in competitive environments.

The remainder of the paper is structured as follows: first, in Section 2, we introduce an assortment optimization and dynamic pricing framework that will guide the development of the model. In Section 3, we describe the model in general terms and quickly move to a number of easier-to-implement heuristics that are compatible with modern airline RM systems. In Section 4, we test these heuristics in the Passenger Origin-Destination Simulator (PODS), a complex airline revenue management simulator that incorporates customer choice and airline competition. Finally, Section 5 concludes and discusses opportunities for future work on customized airline RM.

2. A Definitional Framework for Customized Assortment Optimization and Dynamic Pricing

2.1. Assortment Optimization and Dynamic Pricing in the Airline RM Problem

Fundamentally, the airline revenue management problem is concerned with determining the set of *fare products* to offer to an arriving passenger at a specific time as a function of a forecast of remaining demand, the remaining capacity on the flight leg, and the time remaining until flight departure. In *leg-based* revenue management, the fare products are typically a set of leg fare classes—which are often labeled with letters, such as Y, B, M, and Q—for each individual flight leg. These fare classes consist of a fixed fare and potentially a set of fare restrictions, such as an advance purchase requirement or a required minimum stay. Revenue-maximization heuristics such as EMSRb (Belobaba, 1992) can then be used to generate a set of *booking limits* for each fare class as a function of a forecast of remaining demand.

In more complex *network* revenue management methods, the set of fare products consists of origin-destination itinerary fares (ODIFs) that cover the set of origin-destination markets. Each ODIF has a fixed fare and could involve travel on multiple flight legs. Using a forecast of market demand, a network optimization algorithm is used to generate *leg bid prices* that capture the marginal value of capacity on each flight leg. These leg bid prices are then used as availability controls to determine whether to accept or reject a booking request for a specific ODIF at a given time.

Note that in both leg-based RM and network RM, the prices of the fare products are taken as exogenous inputs that do not change. That is, traditional airline RM focuses on *assortment optimization* (AO)—selecting the set of products to offer a customer at a given time. The classic assortment optimization problem is well-represented in the operations management literature: firms facing a constraint (such as available display space on a shelf) need to determine the optimal assortment, or set of products, to display to a customer to maximize revenue, given some assumption about the customer's choice function. These models can be made more complex by the introduction of limited, perishable inventories, stock-outs, strategic customers, different customer arrival or demand models, or product substitution. Kök and Fisher (2007) provide a good overview of recent work in the assortment optimization literature.

Note the commonalities between the assortment optimization problem and the airline RM framework. In most airline RM models, airlines are deciding which of a series of fare products (for instance, fare classes $\{Y, M, B, Q, X\}$) to offer to a customer at a specific moment in time. Using either booking limits or bid-price control, an assortment of available fare products (perhaps $\{Y, M, B\}$) is displayed to the customer. Talluri and van Ryzin (2004a) provides a detailed method for creating so-called "efficient sets" of products to offer when information about the customer choice function is known. Yet in Talluri and van Ryzin (2004a), as in other airline RM models, the prices of these products do not change.

Despite the exogeneity of prices in traditional airline RM models, many academics and practitioners have discussed the airline revenue management problem using the language of "dynamic pricing." One of the earliest seminal papers on the multiproduct pricing problem described the issue as such: “Given an initial inventory of items and a finite horizon over which sales are allowed, we are concerned with the tactical problem of dynamically pricing the items to maximize total revenue” (Gallego and van Ryzin, 1994, 1997). Here, firms aim to select a vector of prices for an (unchanging) set of products that maximize expected revenue, given a demand function for each product. Unlike Talluri and van Ryzin (2004a), the set of products that are offered to the customer does not change. More recently, scientists from IT vendor Amadeus have described dynamic pricing in a more nuanced way: “Dynamic [calculation of] the optimal price, taking into account the airline’s strategy, customer-specific information, and real-time alternative offerings” (Fiig, 2015). This is a more involved definition that also incorporates knowledge of competitors and customers.

While traditional airline RM seems to fit more closely into the assortment optimization context, advancements in distribution technology could make dynamic pricing possible in future RM systems. With these examples in mind, we propose the following definitions for assortment optimization and dynamic pricing that will be useful as we formulate our model:

Definition:

- Firms practice ***assortment optimization*** when they offer *different sets of products (with static prices) to different customers*, as a function of an observable state of nature.
- Firms practice ***dynamic pricing*** when they charge *different customers different prices for the same set of products*, as a function of an observable state of nature.
- Firms practice ***assortment optimization with dynamic pricing*** when they offer *different sets of products to different customers, then choose the prices for these products*, as a function of an observable state of nature.

In this definition, the *observable state of nature* could include inventory remaining, time remaining in the selling process, characteristics of the customer, assumptions of demand-to-come, and/or competitor offerings.

Depending on the components of the observable state of nature, existing RM optimization models could be classified into this framework. For instance, consider traditional airline RM models with *restricted fare structures*, in which fare classes are differentiated through fare restrictions such as advance purchase requirements, minimum stays, or nonrefundability. With restricted fare structures, both leg-based heuristics like EMSRb (Belobaba, 1992) and network heuristics like displacement-adjusted virtual nesting (DAVN) (Smith and Penn, 1988) can be seen as *assortment optimization* problems, as they are concerned with determining availability for distinct products with fixed prices. Choice-based RM—a theoretical framework that incorporates customer choice into RM optimization (Talluri and van Ryzin, 2004a; Liu and van Ryzin, 2008; Miranda Bront et al., 2008), as well as recent work on personalized assortment generation (Golrezaei et al., 2014; Bernstein et al., 2015; Gallego et al., 2016) can also be defined as assortment optimization problems.

In contrast, multiproduct pricing problems (Gallego and van Ryzin, 1994, 1997) or airline revenue management in fully-unrestricted fare environments where fare products are not differentiated by restrictions such as minimum stays or nonrefundability could be seen as *dynamic pricing* problems in this framework. Fewer papers have started to consider the combination of assortment optimization with dynamic pricing; one notable exception is Besbes and Sauré (2016), who consider a two-firm model that combines both pricing and product selection. The NDC "personalized offer" concept also fits clearly into the *assortment optimization with dynamic pricing* category.

2.2. Personalized assortment models

Before proposing our approach for customized airline revenue management under this assortment optimization/dynamic pricing framework, it is worthwhile to examine a series of recent papers that has begun to formulate "personalized" assortment optimization problems. In these models, the *state of nature* that is considered by the model includes some observation of customer characteristics. Typically, these customer characteristics inform the identification of a choice function, which then determines the products that are made available in the assortment.

In the economics literature, personalized pricing has typically been shown to intensify competition and reduce revenues (Thisse and Vives, 1988). However, through the use of targeted offers or personalized promotions (Chen and Iyer, 2002; Shaffer and Zhang, 2002), some economists have found that personalized pricing can lead to revenue increases in some situations. These models typically do not consider perishable products or products where inventory is limited.

In the operations research (OR) literature, personalization has only recently started to emerge as a key theme. Aydin and Ziya (2009) provide one of the first OR applications of personalization in dynamic pricing for a problem with limited inventories. In their model, customers signal their identity as one of two types, and the firm charges a personalized price based on its interpretation of this signal. The Aydin and Ziya (2009) model considers only two customer types in a monopolistic environment, and does not model the effect of competition.

Following work by Rusmevichientong et al. (2010), Golrezaei et al. (2014) proposed a more flexible approach to personalized assortment optimization. Similar to Aydin and Ziya (2009), customers make a signal to the firm that contains some (potentially noisy) information about that customer's type. The firms can then use this information to create a customized assortment of products. Their model allows for stockouts, but assumes a fixed capacity of products that is set in advance—not the case in the airline revenue management environment, in which heuristics are used to select the number of seats to be offered in each fare class, thereby changing the assortment of products offered to the customer. Bernstein et al. (2015) also consider customer-specific personalized assortment optimization, but the products in their model are of the same price. Chen et al. (2015a) consider a related dynamic pricing problem in which inventories are limited, but customer types do not vary. Federgruen and Hu (2015) and Abeliuk et al. (2016) also consider assortment optimization in contexts where customers are of various types.

Three very recent papers have advanced the work on the personalized assortment optimization problem with customer type as part of the observable state of nature. Chen et al. (2015b) expand the work of Golrezaei et al. (2014) to create an algorithm to construct decision policies based on past observations of customer arrivals and customer choice. They provide an airline example, but only for an ancillary product (priority seating) and not for a series of products in a fare structure as in the traditional airline revenue management problem.

Besbes and Sauré (2016) provide an attractive exploration of a competitive model where firms compete in a duopoly using both the assortment optimization and assortment optimization with dynamic pricing frameworks. They find Nash equilibrium outcomes in the cases when the sets of available products from the two firms do not overlap. Differences in customer choice functions by type are considered. The paper is one of the first to look at competition in assortment optimization, but it is not clear how their theoretical results would extend to the airline RM problem.

Finally, the model proposed by Gallego et al. (2016) is perhaps closest to our model. Their model of personalized assortment optimization is based on the choice-based revenue management frameworks of Talluri and van Ryzin (2004a) and Gallego et al. (2004). Upon observing a customer's type k, the airline uses an assortment selection algorithm Π to compute an assortment $S_k^{\Pi}(c,t)$ as a function of the time t and the capacity remaining c. The airline then faces a value function $V^{\Pi}(c,t)$ with the following Bellman equation:

$$\frac{\partial V^{\Pi}(c,t)}{\partial t} = -\sum_{k=1}^{K} \lambda_k(t) \sum_{n=1}^{N} E[P^k(n, S_k^{\Pi}(c,t))][r_n - \Delta_{l(n)} V^{\Pi}(c,t)] \tag{1}$$

As in Golrezaei et al. (2014), the Gallego et al. (2016) model generates probabilities that a customer of type k is shown assortment S when arriving at time t. However, note that this value function requires a number of inputs that would be difficult to estimate in reality. They include $\lambda_k(t)$, the probability that a customer of type k arrives at time t, and $P^k(n, S_k^{\Pi}(c,t))$, which is the probability that a customer of type k purchases product n from assortment S. The authors acknowledge that this value function "could be very difficult to solve in practice" and propose a number of heuristics to generate the assortments.

The main differences between our approach and Gallego et al. (2016) are (1) our model applies customization *after* optimization by a traditional airline RM system, requiring fewer changes to the underlying models that drive RM practices; (2) our model allows for network revenue management approaches where individual itinerary products can use multiple flight leg resources; (3) our model allows for competitor assortments to be included in the customer choice function; (4) our model does not require a forecast of type-by-type arrival rates; and (5) our model also allows for the possibility of changing the prices of offered products, along with the assortments of products that are offered to the customer.

In the next section, we introduce our general model for formulating customized airline RM. We also propose several heuristics, which we term "probabilistic dynamic availability" (PDynA), that can adjust either the assortment of fare products offered or the prices of those products depending on an arriving customer's type. These heuristics are directly compatible with many revenue management models and systems used by airlines today, as well as future approaches that will be enabled by the New Distribution Capability.

3. Formulating the Customized Airline Revenue Management Problem

In this section, we provide a general model for formulating the customized airline revenue management problem using a assortment optimization and dynamic pricing framework. After describing the model under a general model of passenger demand, we move to the development of several heuristics using a specific demand model in which passengers purchase the lowest cost option[1] out of the sets of itineraries presented to them, subject to an out-of-pocket willingness-to-pay (WTP) budget constraint. These "dynamic availability" heuristics, which are extensions of methods first proposed in Wittman and Belobaba (2016a,b), are tested in the PODS revenue management simulator in Section 4.

3.1. Model primitives

The general flow of the model proceeds as follows: first, an existing revenue management optimization system is used to generate either booking limits (in the case of leg-based revenue management) or bid prices (in the case of network revenue management). These availability controls are used to create an assortment of fare products which would ordinarily be displayed to the passenger. At this point, information about customer characteristics is observed or inferred from the customer's behavior. As a function of this information, the model then considers whether to add or sub-

[1]Here, cost could be described either in terms of out-of-pocket cost or *generalized cost* (Proussaloglu and Koppelman, 1999; Fiig et al., 2010) that also takes into account characteristics of the itinerary, such as the total travel time, circuity, number of stops, or seating options.

tract products from the offered assortment, or to change the prices of any of the products in the assortment. A (potentially) modified assortment is then presented to the customer.

Let $\mathcal{L} = \{1, \ldots, \ell \ldots, L\}$ represent the flight legs in the airline's network, and $\mathcal{C} = \{c_1, \ldots, c_\ell \ldots, c_L\}$ represent the capacities of those flight legs. Then, define $\mathcal{F} = \{1, \ldots, k, \ldots N\}$ as the set of *itinerary products*[2] that are sold by the airline. Each itinerary consists of a vector of one or more flight legs utilized for that itinerary product. For instance, for a passenger flying from Boston to Athens via Paris, $k = \{123, 456\}$ could represent an itinerary that consists of Flight 123 from Boston to Paris and Flight 456 from Paris to Athens in a particular fare class. Each itinerary product $k \in \mathcal{F}$ is also associated with a fare $f_k \in \mathcal{P} = \{f_1, \ldots, f_k, \ldots, f_N\}$.

Observe that there could be multiple *itinerary products* associated with a given airline *itinerary.* As a simple example, consider a nonstop itinerary on Flight 123 from Boston to Paris. This nonstop itinerary could be sold in a number of different fare classes (for instance, classes Y, M, B, and Q), each of which correspond to a set of fare restrictions and a unique price. In this example, each of classes Y, M, B, and Q in the non-stop market from Boston to Paris on Flight 123 would be a unique itinerary product k with a unique price f_k. That is, the *itinerary product* concept is intended to model the one-to-many mapping between itinerary routings and the various conditions (prices and fare restrictions) with which the itineraries are sold in airline distribution systems.

Finally, let m represent a origin-destination market for air transportation (for instance, Boston – Paris and Boston – Athens would both be air transportation markets), and define $\mathcal{F}_m$ as the set of all itinerary products serving market m. An *assortment* $S \subseteq (\mathcal{F}_m \cup \{\emptyset\})$ is then defined as a selection of zero or more itinerary products in $\mathcal{F}_m$.

Let $w \in W$ index the type of a given customer. In our examples that we simulate in Section 4, W consists of only two types {Leisure, Business}, but the cardinality of this set could be extended without loss of generality. Then, suppose that an airline i offers an assortment S in a given market m, and its competitor(s) offer a (joint) assortment R. A customer of type w has probability

[2] In the airline industry, the concept of "itinerary product" is also referred to as an "origin-destination itinerary fare" (ODIF) (Talluri and van Ryzin, 2005)

$P_w(k; f_k|S, R)$ of purchasing itinerary product k given the offered assortment S with competitor assortment R, where $P_w(k; f_k|S, R) = 0$ if $k \notin S \cup R$. Note that this probability function $P_w(\cdot|\cdot)$ is left as a highly general consumer choice model that also (optionally) takes into account competition. Several popular consumer choice models, including the multinomial logit (MNL) model, could fit into this framework.

3.2. Customized airline revenue management

We will first reformulate the traditional airline revenue management problem with the model primitives described in the previous section. We proceed using a network revenue management formulation using dynamic programming (Lautenbacher and Stidham, 1999), but any network or leg-based RM method could also be used.

3.2.1. Stage 1: Generate Bid Prices or Itinerary Product Booking Limits

In this stage, at a given time t, we generate either leg bid prices $\pi_{\ell,t}$ or itinerary product booking limits $b_{k,t}$ for flight legs or itinerary products. These values could be generated using any model in the voluminous literature regarding non-customized airline RM. As an example, the Lautenbacher and Stidham (1999) dynamic programming approach to network RM uses the following leg value function:

$$V_{\ell,t} = \sum_{k \in \mathcal{F}:\ell \in k} \lambda_{k,t} \cdot \max\{f_k + V_{\ell,t+1}(x_l + 1), V_{\ell,t+1}(x_l)\} + \lambda_{0,t} V_{\ell,t+1}(x_\ell) \tag{2}$$

where x_ℓ represents the number of bookings on leg ℓ, $\lambda_{k,t}$ is the forecast demand for itinerary product k at time t,[3] and $\lambda_{0,t}$ represents the probability of no arrival. Using these value functions, we can compute the leg bid prices $\pi_{\ell,t}(x_\ell) = V_{\ell,t}(x_\ell) - V_{\ell,t}(x_\ell + 1)$. This stage ends when the leg bid prices $\pi_{\ell,t}$ (or alternatively the booking limits $b_{k,t}$ if using a leg-based RM heuristic) have been computed for each leg $\ell \in L$ or product $k \in \mathcal{F}$.

[3]If the time periods t were short enough to allow at most one arrival, the quantity $\lambda_{k,t}$ could represent an arrival rate for demand for itinerary product k

3.2.2. Stage 2: Assortment Selection

Given the leg bid prices or product booking limits computed in the previous stage, the RM system next selects an assortment S^* of itinerary products k that meet certain conditions. For instance, in network RM with additive bid price control (Talluri and van Ryzin, 1998; Bertsimas and Popescu, 2003), the fare f_k of any offered itinerary product k must exceed the sum of the leg bid prices $\pi_{\ell,k}$ for all $\ell \in k$. The optimization problem to select S^* then becomes:

$$S^* = \arg\max_{S \subseteq \mathcal{F}_m} \sum_{k \in S} f_k \tag{3}$$

Subject to:

$$f_k \geq \sum_{\ell \in k} \pi_{\ell,t}(x_\ell) \qquad \forall k \in S^*$$

$$c_\ell \geq 1 \qquad \forall \ell \in k, \forall k \in S^*$$

With leg-based EMSRb, an itinerary product k would be included in assortment S^* as long as its associated booking limit $b_{k,t} \geq 1$ and there is at least one seat available on each leg used in the itinerary product (Belobaba, 1992). Note that Stages 1 and 2 are simply a reformulation of the traditional airline RM problem in assortment optimization language. The assortment S^* is the assortment of fare products that is ordinarily offered by existing RM systems of the types used by major airlines worldwide.

3.2.3. Stage 3: Passenger Arrival

At this stage, a passenger arrives to book in a market m, and we assume that the airline can observe the customer's type w. In practice, this customer segmentation into w-types could occur through self-identification through a log-in or frequent flier number, observation of past trip purchases, itinerary attributes, the characteristics of the request, data mining, social media, or other methods. We do not specifically discuss type segmentation in this paper, but refer readers to a growing literature in data mining and customer profile-building (e.g. Adomavicius and Tuzhilin (2001); Liao et al. (2012); He et al. (2013) and many others).

After the customer's type w is observed or inferred, we assume that this type observation allows the airline to estimate the customer's choice function $P_w(k; f_k|S, R)$. Note that this is the first time that the passenger's type is observed in the model (that is, only after the initial optimization and selection of assortment S^*). Moreover, all demand forecasts $\lambda_{k,t}$ in the previous stage of the model were made without knowledge of passenger type.[4]

3.2.4. Stage 4: Dynamic Availability

We are now ready to answer the key question of customized airline revenue management: should we make any adjustments to the assortment S^* or the price vector $P^* = \{f_k : k \in S^*\}$ after observing the customer's type w? We propose two different mechanisms to answer this question: first, we could add or remove itinerary products from the assortment S^*. This could be accomplished by performing column generation to search for a new assortment with positive reduced cost (Gallego et al., 2016).

Alternatively, we could adjust the prices in the assortment price vector P^*. In an optimization context, this is equivalent to adjusting the reduced costs of the itinerary products in the existing assortment. Note that while the first method takes an assortment optimization approach to customized airline RM, the second uses a dynamic pricing approach. Once any adjustments are made, a modified custom assortment $S^{*'}$ or a modified custom price vector $P^{*'}$ is offered to the passenger.

Customized assortment optimization

To generate a custom assortment $S^{*'}$, first define candidate assortments S_j for all $j \in \mathcal{F}_m$ as follows:

$$S_j = \begin{cases} S^* \cup j & \text{if } j \notin S^* \\ S^* \setminus j & \text{if } j \in S^* \end{cases}$$

Then, generate a custom assortment $S^{*'}$ by searching among the candidate assortments S_j for the

[4]In today's airline revenue management systems, forecasts are commonly made on the itinerary product (ODIF) level, but not on the customer type-ODIF level.

one that maximizes the expected myopic revenue gain:

$$S^{*'} = \arg \max_{S' \in \{S_j \cup S^*\}} \left(\sum_{k \in S'} f_k P_w(k; f_k | S', R) - \sum_{k \in S^*} f_k P_w(k; f_k | S^*, R) \right) \tag{4}$$

Subject to:

$$c_l \geq 1 \qquad \forall l \in k, \forall k \in S'$$

That is, a new assortment $S^{*'}$ is generated by either adding or subtracting a single itinerary product from the original assortment S^*. Itinerary products are added or removed from S^* if doing so would increase the expected revenue of offering the revised assortment $S^{*'}$ to customer type w.

This process could be iterated to add or subtract multiple itinerary products from the original assortment S^*; after the first custom assortment $S^{*'}$ is generated, a new set of candidate assortments could be constructed. A new custom assortment $S^{*''}$ could then be generated; this process would repeat until the algorithm does not choose to add or subtract any products. Note that in this stage of the method, bid price controls and/or booking limit restrictions are not enforced; decision rule (4) is a greedy heuristic that searches for the custom assortment that will generate the largest myopic increase in revenue given the observation of the passenger's type.

Adjustments to the price vector: customized dynamic pricing

Instead of changing the itinerary products in assortment S^*, we could also decide to hold the assortment fixed and instead change the vector of prices $\mathcal{P}^* = \{f_k : k \in S^*\}$ for itinerary products in S^*. To do so, we will compute a price adjustment vector Δ^* as follows:

$$\Delta^* = \{\Delta_k : \arg \max_{\Delta_k \in [l_k, u_k]} \sum_{k \in S^*} (f_k + \Delta_k) P_w(k; (f_k + \Delta_k) | S^*, R) \qquad \forall k \in S^*\} \tag{5}$$

The price adjustment vector is constructed by jointly determining the best increments or decrements for the price of each itinerary product in S^*. Then, the final adjusted price vector $\mathcal{P}^{*'}$ is computed by a piecewise addition $\mathcal{P}^{*'} = \{f_k + \Delta_k^* : k \in S^*\}$ for each itinerary product in S^*.

Note that the formulation of the price adjustment vector Δ^* allows for the individual itinerary product price adjustments to be bounded in an interval $[l_k, u_k]$. While this interval could be set to

$(-\infty, \infty)$ to allow for the most flexibility in adjusting prices, it may be desirable due to market or competitor conditions to bound the adjustments by smaller ranges (for instance, the final adjusted prices could be bounded by the highest and lowest original prices of the itinerary products in S^*, or by the gap in prices between the next highest or lowest priced product). These bounds are discussed in more detail in Section 4.

3.3. "Last-class" heuristics

Note from the previous section that selecting the optimal customized assortment $S^{*'}$ or customized price vector $\mathcal{P}^{*'}$ requires the computation of many probabilities $P_w(k; f_k|S, R)$. In cases in which it is not possible to generate a good estimate of $P_w(\cdot|\cdot)$, or when the number of itinerary products k in the initial assortment is very large, considering every possible combination of products to add or remove or prices to change may be unwieldy in practice. For instance, for a leisure passenger who is likely to purchase the least-expensive available fare class for a given itinerary, it is likely not necessary to consider whether or not to remove the most expensive itinerary product from the assortment S^*, as it is unlikely that the customer would ever purchase that product.

Customers searching for airline tickets are often shown the resulting itineraries in increasing order of price, with the least expensive itinerary displayed first. In this situation, many customers will typically choose to purchase the least expensive option. These passengers have been commonly referred to as "priceable" demand (Boyd and Kallesen, 2004) or "price-oriented" demand (Fiig et al., 2010; Lapp and Weatherford, 2014). While many leisure passengers could fit into the "price-oriented demand" category, an increasing number of business passengers facing increasingly-restrictive corporate travel policies that require the purchase of the least expensive available itinerary (Mouawad, 2012) could also be qualified as "price-oriented." For price-oriented demand, we consider restricting our customized adjustments to the least-expensive itinerary product in the assortment S^*. For a single non-stop itinerary, this is equivalent to limiting our attention to the price and availability of the lowest open fare class on that flight leg. Hence, we call these strategies *last-class heuristics*.

Without loss of generality, reorder the itinerary products in $\mathcal{F}_m = \{1, \ldots, k, \ldots, n\}$ in descending order of price such that $f_i \geq f_j \; \forall i \leq j$. Suppose that itinerary products $\{1, \ldots, k\}$ are offered in assortment S^*, and itinerary products $\{k+1, \ldots, n\}$ are not offered in the assortment.

In last-class assortment optimization, we decide whether to subtract or add a single product to the assortment S^*. In a nonstop market with a single itinerary available, this is equivalent to deciding whether to close the current available class or to open availability in one class below the current available class. Wittman and Belobaba (2016a) referred to a simplier implementation of these actions as "class-based dynamic availability" (DynA).

"Last-class" assortment optimization:

Construct two candidate assortments: $S^+ = S^* \cup \{k+1\}$ and $S^- = S^* \setminus k$.

Then, offer assortment S^+ if:

$$\sum_{k \in S^+} f_k P_w(k; f_k | S^+, R) - \sum_{k \in S^*} f_k P_w(k; f_k | S^*, R) > 0 \tag{6}$$

Offer assortment S^- if:

$$\sum_{k \in S^-} f_k P_w(k; f_k | S^-, R) - \sum_{k \in S^*} f_k P_w(k; f_k | S^*, R) > 0 \tag{7}$$

Example: Suppose individual passengers i of type w in market m have a random *willingness-to-pay budget* $\theta_i \sim \Theta_{w,m}$, where $\Theta_{w,m}$ is a probability distribution specific to type w and market m. Consumers select a single itinerary product k that maximizes their consumer surplus $V_i = \theta_i - f_k$, or decide to purchase nothing. In a single-carrier market, this is equivalent to the following choice function:

$$P_w(k; f_k | S, \emptyset) = \begin{cases} 1 & \text{if } f_k < f_j \ \forall j \in S \text{ and } f_k \leq \theta_i \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

With choice model (8), passengers always purchase the least-expensive itinerary product they are offered, as long as the fare is lower than their willingness-to-pay budget θ_i. This choice model leads to a simple customization heuristic.

Proposition 1 (Probabilistic DynA):

Under choice model (8), offering assortment $S^+ = S^* \cup \{k+1\}$ will lead to higher myopic expected revenues than offering assortment S^* if::

$$f_{k+1} \cdot \text{Prob}(\theta_i \geq f_{k+1}) > f_k \cdot \text{Prob}(\theta_i \geq f_k) \tag{9}$$

Offering assortment $S^- = S^* \setminus k$ will lead to higher myopic expected revenues than offering assortment S^* if:

$$f_{k-1} \cdot \text{Prob}(\theta_i \geq f_{k-1}) > f_k \cdot \text{Prob}(\theta_i \geq f_k) \tag{10}$$

That is, itinerary product $k+1$ is added to the assortment if its expected marginal revenue exceeds the expected marginal revenue from itinerary product k. Similarly, if the expected marginal revenue of itinerary product $k-1$ exceeds the expected marginal revenue from itinerary product k, k is removed from the assortment and a custom assortment S^- is offered. It should be noted that this is a *myopic* algorithm insofar as it will always value gaining a new booking as opposed to withholding inventory to satisfy future potential demand.

Proof (Proposition 1):

Under choice model (8), customers facing assortment S^* will buy product k if $\theta_i \geq f_k$ and purchase no product otherwise. Facing assortment $S^+ = S^* \cup \{k+1\}$, customers will buy product $k+1$ if $\theta_i \geq f_{k+1}$ and buy no product otherwise.

Therefore, the expected revenue from offering assortment S^* is $[f_k \cdot \text{Prob}(\theta_i \geq f_k)]$, and the expected revenue from offering the larger assortment S^+ is $[f_{k+1} \cdot \text{Prob}(\theta_i \geq f_{k+1})]$. Therefore, offering assortment S^+ will result in higher expected revenues than offering assortment S^* if:

$$f_{k+1} \cdot \text{Prob}(\theta_i \geq f_{k+1}) > f_k \cdot \text{Prob}(\theta_i \geq f_k)$$

Similarly, the expected revenue from offering the smaller assortment $S^- = S^* \setminus k$ is $[f_{k-1} \cdot \text{Prob}(\theta_i \geq$

$f_{k-1})]$, and the expected revenues for offering S^- will exceed those from offering S^* if:

$$f_{k-1} \cdot \text{Prob}(\theta_i \geq f_{k-1}) > f_k \cdot \text{Prob}(\theta_i \geq f_k)$$

□

Alternatively, we could consider adjusting the price of the least expensive itinerary product without changing availability. In last-class dynamic pricing, we either increment or decrement the fare of the least expensive itinerary product by a value Δ_m^* bounded by a selected interval $[l, u]$. Wittman and Belobaba (2016b) refer to this type of strategy as "fare-based dynamic availability."

"Last-class" dynamic pricing:

Select a fare modification Δ_k^* such that:

$$\Delta_k^* = \arg \max_{\Delta_k^* \in [l,u]} \sum_{i=1}^{k-1} f_i P_w(i; f_i | S^*, R) + (f_k + \Delta_k^*) P_w(k; (f_k + \Delta_k) | S^*, R) \tag{11}$$

Using choice model (8), we can then create a decision rule for practicing last-class dynamic pricing.

Proposition 2 (Probabilistic FDynA):

To prevent fare inversions, let $l_k = (f_{k+1} - f_k) \leq 0$ and $u_k = (f_{k-1} - f_k) \geq 0$. Then, under choice model (8), the last-class dynamic pricing heuristic (11) reduces to:

$$\Delta_k^* = \arg \max_{\Delta_k^* \in [l_k, u_k]} (f_k + \Delta_k^*) \cdot \text{Prob}(\theta_i \geq (f_k + \Delta_k^*)) \tag{12}$$

Proof (Proposition 2):

Under choice model (8), as long as $f_k < f_{k-1}$, customers will either purchase product k or purchase nothing. Therefore,

$$\sum_{i=1}^{k-1} f_i P_w(i; f_i | S^*, R) = 0$$

Then, under choice model (8),

$$P_w(k; (f_k + \Delta_k^*) | S^*, R) = \text{Prob}(\theta_i \geq (f_k + \Delta_k^*))$$

Hence, the last-class dynamic pricing heuristic (11) reduces to:

$$\Delta_k^* = \arg \max_{\Delta_k^* \in [l,u]} [(f_k + \Delta_k^*) \cdot \text{Prob}(\theta_i \geq (f_k + \Delta_k^*))]$$

$\square$

3.4. Estimating choice probabilities

Note that if the passenger choice models $P_w(k; f_k|S, R)$ can be estimated with confidence, the decision rules (6), (7), and (11) can be used directly to compute the optimal custom assortment $S^{*'}$ or the optimal price vector $P^{*'}$. Some practitioners and researchers have estimated these types of choice probabilities in past work, often using the multinomial logit model (Ratliff and Gallego, 2013; Johnson et al., 2014; Carrier and Weatherford, 2014; Lurkin et al., 2016).

As mentioned earlier, estimating $P_w(k; f_k|S, R)$ could be challenging, particularly in competitive markets (Collins et al., 2013). With the advancement of fast-response internet search engines, the "look-to-book ratio" (the ratio of product searches to actual purchase decisions) continues to rise (Fiig et al., 2015). The prevalance of "no-purchase" observations could skew the estimation of the choice probabilities $P_w(k; f_k|S, R)$.

However, assuming a simple price-oriented choice model (8), the simpler decision rules (9), (10), and (12) can instead be used. Note that with these decision rules, it is not necessary to estimate $P_w(k; f_k|S, R)$. Instead, airlines need to estimate the willingness-to-pay (WTP) budget distributions $\Theta_{w,m}$ such that $\text{Prob}(\theta_i \geq f_k)$ can be computed for passengers of type w in market m. While this is still a complex problem to solve in practice, it requires less information than estimating $P_w(k; f_k|S, R)$. Furthermore, techniques for estimating passenger WTP are currently in development in the academic and industrial literature (e.g. Seelhorst and Liu (2015)).

Wittman and Belobaba (2016b) proposed a simple WTP estimation method that could be used to guess passenger WTP in a given market. In their approach, a "Q-multiplier" for each market m and passenger type w is estimated. Then, the mean WTP is computed as follows:

$$E[\Theta_{w,m}] = QMULT_{w,m} \cdot f_n$$

where f_n is the least expensive itinerary product in $\mathcal{F}_m$, irrespective of availability. In Wittman and Belobaba (2016b), this estimate $E[\Theta_{w,m}]$ was used as a point estimate of mean passenger WTP when adjusting availability or fares. For this probabilistic model, however, the Q-multiplier concept could be expanded into a probability distribution as follows: suppose by the Central Limit Theorem that passenger WTP budgets are distributed by a Gaussian distribution, and assume a coefficient of variation $\gamma_{p,m} = \frac{\sigma}{\mu}$. Then,

$$\text{Prob}(\theta_i \geq f_k) = 1 - \Phi\left(f_k | \mu = E[\Theta_{w,m}], \sigma = (\gamma_{p,m} \cdot E[\Theta_{w,m}]\right)) \tag{13}$$

That is, we assume that passenger WTP budgets are distributed with a Gaussian distribution $N(\mu_{w,m}, \sigma_{w,m})$ where $\mu_{w,m} = QMULT_{w,m} \cdot f_n$ and $\sigma = \gamma_{w,m} \cdot \mu_{w,m})$. This Gaussian distribution can then be used to evaluate the PDynA and PFDynA heuristics (9), (10), and (12).

It is worth recognizing at this point that we have made quite a number of simplifying assumptions from the original assortment optimization and dynamic pricing formulations (4) and (5) to create our probabilistic DynA heuristics (9), (10), and (12). To create our last-class heuristics, we have limited our adjustments to the least-expensive available itinerary products in the assortment S^*. In choice model (8), we assumed a single-carrier market and price-oriented demand. And in estimating θ_i in (13), we assumed a simple Gaussian distribution for passenger WTP budgets.

Yet as we show in the next section, the heuristics (9), (10), and (12) perform quite well even when exposed to an environment that violates these assumptions. As airlines become more adept in estimating the choice probabilities $P_w(k; f_k|S, R)$, the implementation of customized airline RM can become closer to the ideal formulations (4) and (5), thereby further improving the revenue impacts of the heuristics.

4. Simulating Customized Airline RM

In this section, we implement and simulate the simplified customized RM heuristics (9), (10), and (12) in the Passenger Origin-Destination Simulator (PODS). PODS was developed in the early 1990s by Boeing Commercial Airplanes, and has continued to be refined in the following two decades to incorporate new airline RM techniques and heuristics (Barnhart et al., 2003). As a highly robust and complex RM simulator, PODS has been used by a number of researchers to evaluate the performance of RM heuristics in a practical environment (e.g. Gorin and Belobaba (2008); Fiig et al. (2010); Carrier and Weatherford (2014); Wittman and Belobaba (2016b)).

4.1. PODS overview, simulation details, and calibration

PODS is an agent-based simulation that models the interactions between passenger choices and airline revenue management methods in a highly complex, competitive airline network. The tests of the last-class heuristics proposed in Section 3 were completed in PODS Network U10.

4.1.1. Network U10: Airline parameters

Network U10 consists of four competing airlines, each with a hub in the central United States. Each airline serves up to 20 cities west of its hub and up to 20 cities east of its hub. Passenger demand on the network flows in one direction from west to east; along with local demand from each of the spoke cities to the hub, passengers can also fly from western spoke cities to eastern spoke cities by connecting via one of the airline's hubs. All together, the airlines in the network operate 442 flight leg departures per day, serving a total of 572 origin-destination (OD) markets. Figure 1 shows that airline hub-and-spoke network that is modeled in PODS Network U10.

Each OD market is priced using a 10-class fare structure. Using the nomenclature from the previous section, for a single non-stop OD market on which one airline operates one flight, the set of itinerary products would be $\mathcal{F}_m = \{1, \ldots, 10\}$ and the associated price vector would be $\mathcal{P}_m = \{f_1, \ldots, f_{10}\}$. Note that $f_{k-1} > f_k > f_{k+1}$ for any itinerary product k in this fare structure. Each fare class is associated with zero or more *fare restrictions*, meant to model the common techniques used by airlines to segment passengers into high-WTP and low-WTP groups. The fare restrictions in PODS correspond to an advance purchase requirement, a roundtrip purchase requirement, a Saturday night stay requirement, a minimum stay requirement and a nonrefundability requirement. Table 1 shows an example fare structure for the DEN–MIA market in PODS Network U10.

As noted in Figure 1, each airline in PODS Network U10 uses a single revenue management method.

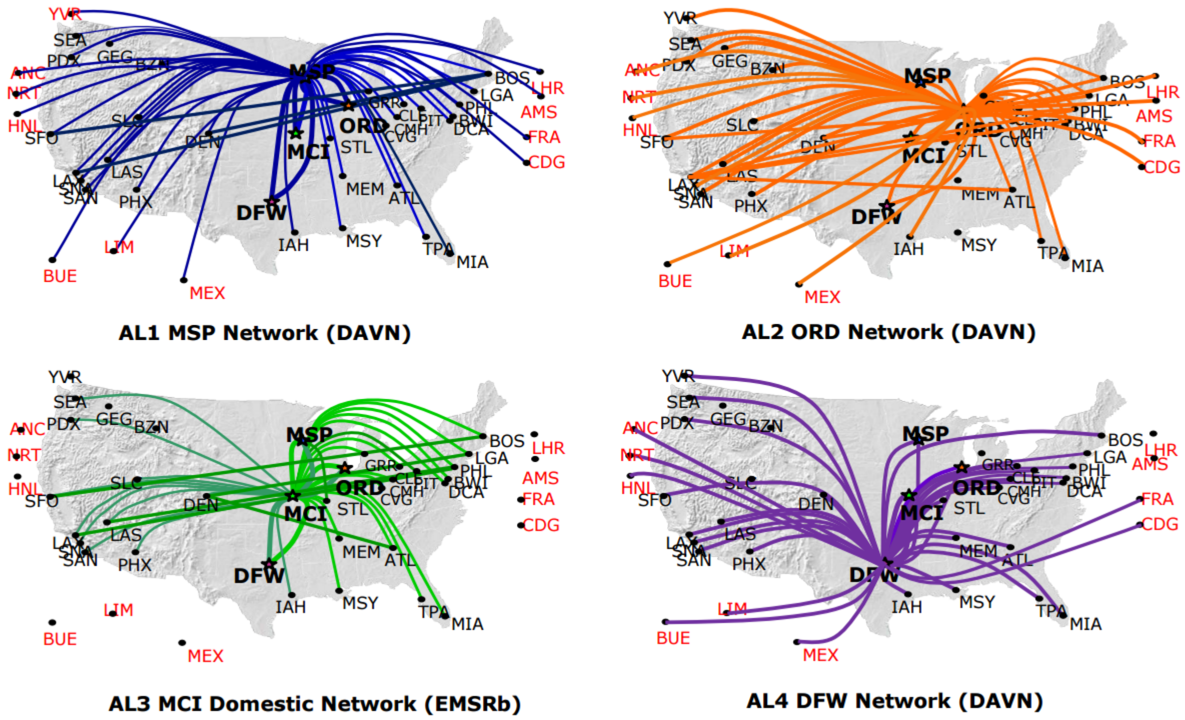

Figure 1: Hub-and-spoke airline networks in PODS Network U10

Class	Fare	Restrictions			
		Adv. Purch.	Min Stay	Nonref.	Sat. Night
1	\$481	0 days	No	No	No
2	\$307	0 days	No	Yes	No
3	\$256	3 days	No	Yes	No
4	\$204	7 days	No	Yes	No
5	\$183	7 days	No	Yes	No
6	\$161	7 days	No	Yes	No
7	\$153	14 days	No	Yes	No
8	\$145	14 days	No	Yes	No
9	\$138	14 days	No	Yes	No
10	\$130	21 days	No	Yes	No

Table 1: Example fare structure for DEN – MIA market in PODS Network U10

Airlines AL1, AL2, and AL4 each use network RM through a method called displacement-adjusted virtual nesting (DAVN) (Smith and Penn, 1988). DAVN is a hybrid network/leg heuristic that uses a network linear program to compute leg bid prices, uses those leg bid prices to generate "displacement-adjusted" fares, and then uses those displacement adjusted fares in a nested structure to generate booking limits using the EMSRb heuristic. That is, DAVN does not use bid price control, but rather generates booking limits for "virtual buckets" consisting of zero or more itinerary products.[5] In contrast, Airline AL3 uses leg-based RM only, applying EMSRb (Belobaba, 1992) to generate leg booking limits for its U.S. domestic network. Each airline uses standard forecasting of class-by-class demand as opposed to more complicated hybrid forecasting or Q-forecasting methods.[6]

4.1.2. Network U10: Passenger parameters

As an agent-based simulation, PODS simulates passenger decision-making by generating a number of individual passengers, endowing these passengers with preferences and decision rules, and allowing the passengers to make choices that maximize their individual utility. Passengers in PODS desire to travel in a single OD market m. After surveying the options presented to her, a passenger will select the itinerary product that she deems to be the best possible option, subject to a budget constraint. If all options exceed her budget constraint, she will decide to "no-go" and not purchase any itinerary product.

At the time of passenger generation, passengers are assigned one of two types $w \in W = \{1, 2\}$. Type $w = 1$ passengers represent business passengers, and Type $w = 2$ passengers represent leisure passengers. Each passenger type is associated with a unique *booking curve* that describes the arrival rates at which passengers of that type arrive to book. Each individual passenger i is also assigned a WTP budget θ_i drawn from a negative exponential distribution $\Theta_{w,m}$. The distribution $\Theta_{w,m}$ is constructed such that all passengers (of either type) are willing to pay at least the lowest fare f_{10} in any given OD market fare structure.

In PODS, $E[\Theta_{1,m}] > E[\Theta_{2,m}]$ for all markets m, reflecting an assumption that business passengers will be willing to pay more, on average, than leisure passengers in any given market. It is important to note that the booking curves, WTP distributions $\Theta_{w,m}$, and individual passenger WTP budgets θ_i are never visible to the airlines in the PODS simulation.

[5] Heuristics using bid-price control, such as Bratu (1998)'s probabilistic bid price (ProBP) method, were also tested with the heuristics described below and produced similar results.

[6] These more complex forecasting methods were also tested with the heuristics described below with similar results.

Along with their WTP budgets, passengers in PODS are also randomly assigned a schedule preference and a series of *disutilities* associated with each of the fare restrictions shown in Table 1. When evaluating itinerary product options, passengers select the itinerary product that minimizes the sum of the itinerary product's out-of-pocket cost f_k, along with the disutilities evaluated for the itinerary product's schedule and associated fare restrictions. The passenger will then purchase this itinerary product if the out-of-pocket cost f_k does not exceed the passenger's WTP budget θ_i.

Note that this choice model represents a departure from the simplistic single-carrier choice model developed in (8). First, the presence of fare restrictions and various schedules, along with heterogeneity of preferences, means that customers will not always buy the least-expensive itinerary product they are offered. Furthermore, the presence of competition in some OD markets means that passengers are not limited to a single airline's set of itinerary products when making their purchase selection.

Despite these significant differences, we will show that heuristics (9), (10), and (12) still perform well in PODS Network U10 under this set of underlying passenger behavior norms. Some researchers (Carrier, 2003; Carrier and Weatherford, 2014) have estimated more complex multinomial logit (MNL) models of customer choice in PODS, although not by customer type; as mentioned in Section 3, better estimates of $P_w(k; f_k|S, R)$ will allow for the heuristics to more closely approach the idealized formulations (4) and (5).

4.1.3. Implementation of customized RM heuristics in PODS

The probabilistic DynA heuristics (9), (10), and (12) were tested in PODS as described in Section 3. First, based on its forecasts of future demand, the airline revenue management system generates booking limits for each itinerary product k. Then, in a given OD market m, an assortment S^* is created. This assortment represents the set of itinerary products that would normally be available in a non-customized RM setting.

At this point, the airline observes (with a certain degree of accuracy) the w-type of an arriving passenger. With this information, the airline then computes its estimate of the passenger's WTP budget distribution $\Theta_{w,m}$ using an *input* Q-multiplier estimate $QMULT_{w,m}$ and an *input* coefficient of variation estimate $\gamma_{w,m}$ as described in Section 3.4. The mean passenger WTP budget is thus estimated to be $\mu_{w,m} = QMULT_{w,m} \cdot f_n$ and the standard deviation is computed as $\sigma_{w,m} = \gamma_{w,m}\mu_{w,m}$. Therefore, $\Theta_{w,m}$ is assumed to be a Gaussian distribution $N(\mu_{w,m}, \sigma_{w,m})$.

With this estimate of the passenger WTP budget distribution $\Theta_{w,m}$, the airline is now ready to perform customized RM. First, consider customized last-class assortment optimization. Suppose that S^* consists of itinerary products $\{1, \ldots, k\}$. If the customer is observed to be of Type $w = 1$ (business), the system uses decision rule (10) to decide whether to remove itinerary product k from the assortment. This is referred to as **Business PDynA**, and is equivalent to "closing" the lowest available fare class for a given itinerary.

If $w = 2$ (leisure), the system uses decision rule (9) to decide whether to add itinerary product $k + 1$ to the assortment.[7]. This is referred to as **Leisure PDynA**, and is equivalent to "opening up" one fare class below the lowest currently available fare class for the itinerary.

Next, consider customized last-class dynamic pricing. For Type $w = 1$ (business) passengers, the system decides whether to *increment* the fare of itinerary product k using decision rule (12) and bounds $\Delta_k^* \in [0, (f_{k-1} - f_k)]$. This method is called **Business PFDynA**. For Type $w = 2$ (leisure) passengers, the system decides whether to *decrement* the fare of itinerary product k using decision rule (12) and bounds $\Delta_k^* \in [(f_{k+1} - f_k), 0]$. This method is called **Leisure PFDynA**. Table 2 reviews the probabilistic dynamic availability heuristics that were implemented in PODS.

Heuristic	Method	Passenger Type	Description
Business PDynA	Assort. Opt.	$w = 1$	Remove product k from S^* if $f_{k-1} \cdot \text{Prob}(\theta_i \geq f_{k-1}) > f_k \cdot \text{Prob}(\theta_i \geq f_k)$
Leisure PDynA	Assort. Opt.	$w = 2$	Add product $k+1$ to S^* if $f_{k+1} \cdot \text{Prob}(\theta_i \geq f_{k+1}) > f_k \cdot \text{Prob}(\theta_i \geq f_k)$
Business PFDynA	Dynamic Pricing	$w = 1$	Increment f_k by $\Delta_k^* = \arg\max_{\Delta_k^* \in [0,(f_{k-1}-f_k)]}$ $(f_k + \Delta_k^*) \cdot \text{Prob}(\theta_i \geq (f_k + \Delta_k^*))$
Leisure PFDynA	Dynamic Pricing	$w = 2$	Decrement f_k by $\Delta_k^* = \arg\max_{\Delta_k^* \in [(f_{k+1}-f_k),0]}$ $(f_k + \Delta_k^*) \cdot \text{Prob}(\theta_i \geq (f_k + \Delta_k^*))$

Table 2: Probabilistic dynamic availability methods tested in PODS

[7] Note that an itinerary product is not added to the assortment if doing so would violate the itinerary product's advance purchase restriction.

The next subsections report the results of the tests of these probabilistic dynamic availability methods in PODS. Note that each PODS "departure day" is simulated 600 times in a series of two trials, with each trial having a distinct random seed. In each trial, the first 200 simulation runs are discarded as a "burn-in period" to allow the airline to build a historical booking database for forecasting purposes. Therefore, results reported in the following sections reflect the average of 800 instances of a single departure day. T-tests show that the reported revenue results have a confidence interval of approximately $\pm 0.05\%$ at the 95% confidence level.

4.2. Results: Probabilistic DynA (Customized assortment optimization)

Note that the probabilistic dynamic availability heuristics displayed in Table 2 rely on two parametric inputs: the Q-multiplier $QMULT_{w,m}$ and the coefficient of variation $\gamma_{w,m}$. Together, these two parameters specify the Gaussian distribution that is used by the airline to estimate the WTP budget distributions for passengers; this in turn allows the airline to compute probabilities $\text{Prob}(\theta_i \geq f_k)$.

4.2.1. Business PDynA

Figure 2 shows the change in Airline 1 (AL1)'s revenue from the base case (in which no dynamic availability heuristic is used) with a variety of Q-multiplier values for type $w = 1$ passengers and a fixed coefficient of variation $\gamma_{w,m} = 0.3$.[8]

As Figure 2 shows, Business PDynA increases AL1's revenues by between 0.15% and 0.55%, depending on the input Q-multiplier value chosen. Recall that as the Q-multiplier increases, the mean of the passenger WTP budget distribution estimate for business passengers also increases. This means that the Business PDynA condition (10) is more likely to be satisfied, particularly when lower (less-expensive) fare classes are included in the original assortment S^*. Therefore, with higher Q-multiplier values, Business PDynA is more likely to remove itinerary product k from the assortment S^*. This relationship is shown in Figure 3; as the Q-multiplier estimate increases, the airline is more likely to remove the least-expensive itinerary product from the assortment offered to the customer.

[8]Changing the coefficient of variation $\gamma_{w,m}$ has negligible effects on the performance of Business and Leisure PDynA.

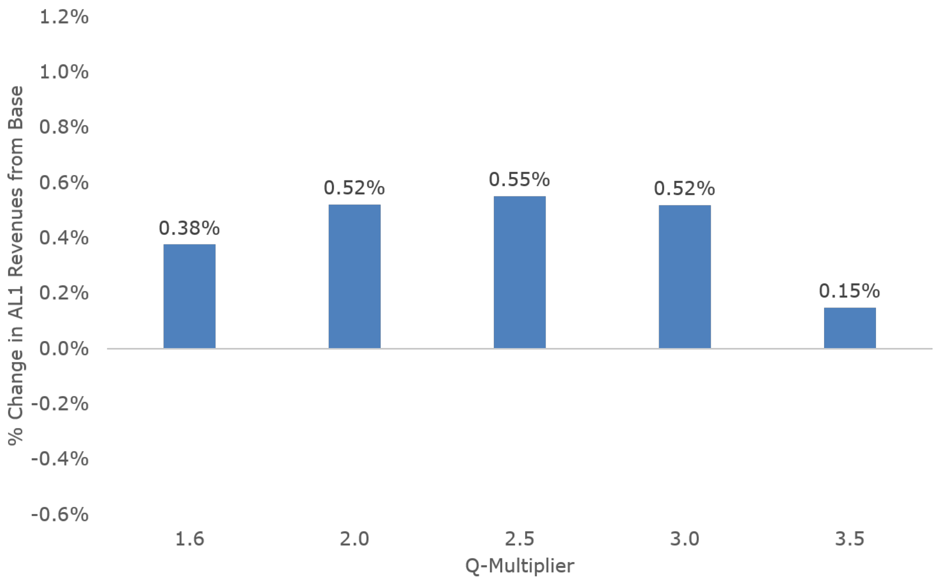

Figure 2: Percent change in AL1 revenues from base when AL1 uses Business PDynA with various Q-multipliers ($\gamma_{w,m} = 0.3$)

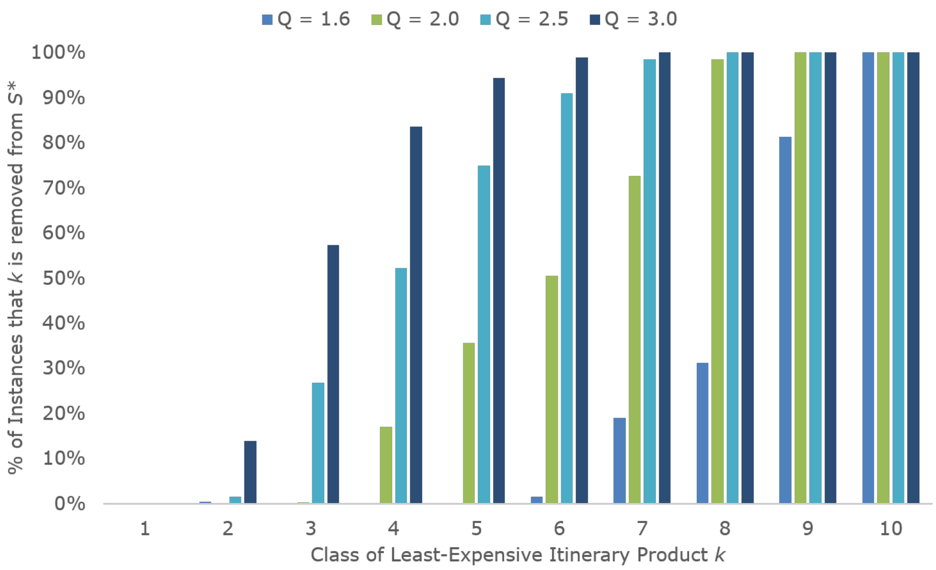

Figure 3: Percent of instances that itinerary product k is removed from S^* with Business PDynA when k is the lowest open class (Various Q-multipliers, $\gamma_{w,m} = 0.3$)

As a result of removing less-expensive itinerary products from the custom assortment $S^{*'}$, AL1's passenger yield—a measure of passenger revenue per revenue-passenger-mile (RPM)—increases as the input Q-multiplier increases from a base of \$0.1267 per RPM to a value of \$0.1286 per

RPM with a Q-multiplier of 2.5. However, AL1's load factor simultaneously decreases from 83.5% to 82.7%; since business passengers are facing customized assortments $S^{*'}$ with more expensive products than the ordinary assortment S^*, these passengers are more likely to either no-go or book with another airline. However, since AL1's revenue increases as a result of Business PDynA, the increase in passenger yield outweighs the overall loss in passengers from using the heuristic.

One of the benefits of the PODS environment is that it allows researchers to examine the effects of one airline's revenue management strategies on the other airlines with which it competes. Figure 4 shows the effects of AL1's use of Business PDynA with a variety of Q-multipliers on each airline's revenues compared to the base case. Note that all airline see revenue gains when AL1 uses Business PDynA; AL1 increases its own yield by removing less-expensive itinerary products from the assortments offered to business passengers, and other airlines gain revenue by capturing business passengers away from AL1's by offering more attractive assortments. The gains in revenues for other airlines increase as AL1's increases its Q-multiplier estimate and as a result is more likely to remove products from assortment S^* for business passengers.

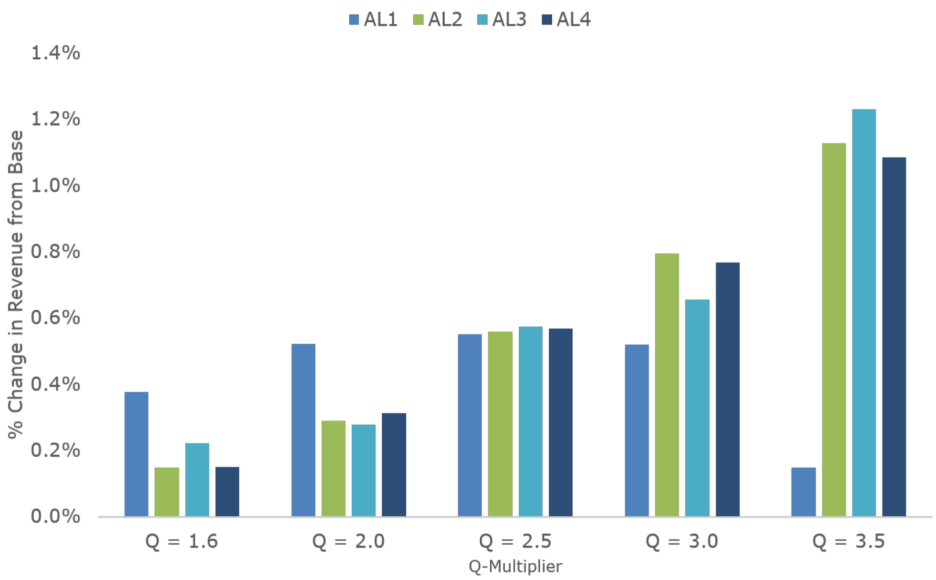

Figure 4: Percent change in revenue from base for all airlines when AL1 uses Business PDynA (Various Q-multipliers, $\gamma_{w,m} = 0.3$)

As could be expected, if all airlines in the simulation practice Business PDynA, revenues increase over the case when only AL1 practices the heuristic. This is because there is no alternative airline option for business passengers to turn to when they face smaller and more-expensive assortments; all airlines are similarly limiting the less-expensive products made available to business passengers. However, we also must consider whether this result is stable in competition; that is, does an airline have an incentive to deviate from the all-airline equilibrium?

As shown in Figure 5, the answer is no; when AL1 deviates from the equilibrium and ceases to practice Business PDynA, revenues for all airlines fall from the all-airline equilibrium case. This suggests that the decreases in passenger yield when AL1 deviates from the equilibrium outweighs the potential gains from attracting business passengers from other airlines by offering more robust assortments. This makes the all-airline scenario for Business PDynA a Nash equilibrium, as no airline has the incentive to change their strategy to practice the heuristic given the actions of the other airlines (Besbes and Sauré, 2016).

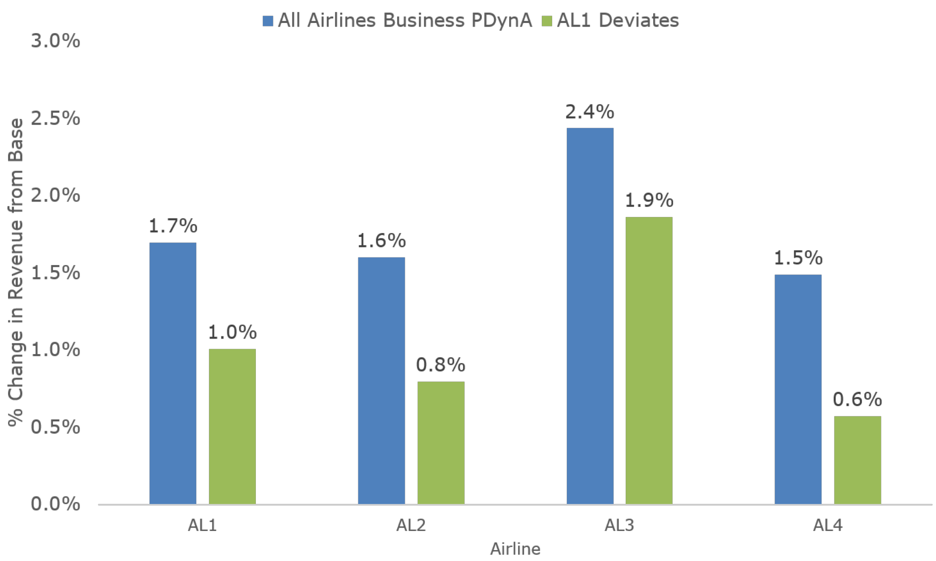

Figure 5: Percent change in revenue from base when various airlines use Business PDynA (Input Q-multiplier = 2.0, $\gamma_{w,m} = 0.3$)

4.2.2. Leisure PDynA

In Leisure PDynA, the airline decides whether to add itinerary product $k+1$ into the assortment S^* for passengers that are identified to be of type $w = 2$. As shown in Figure 6, however, Leisure PDynA strategies are essentially revenue neutral when practiced by AL1 at a variety of Q-multipliers and coefficients of variation γ. The revenue results from this heuristic are generally not significantly different from zero at the 95% confidence level.

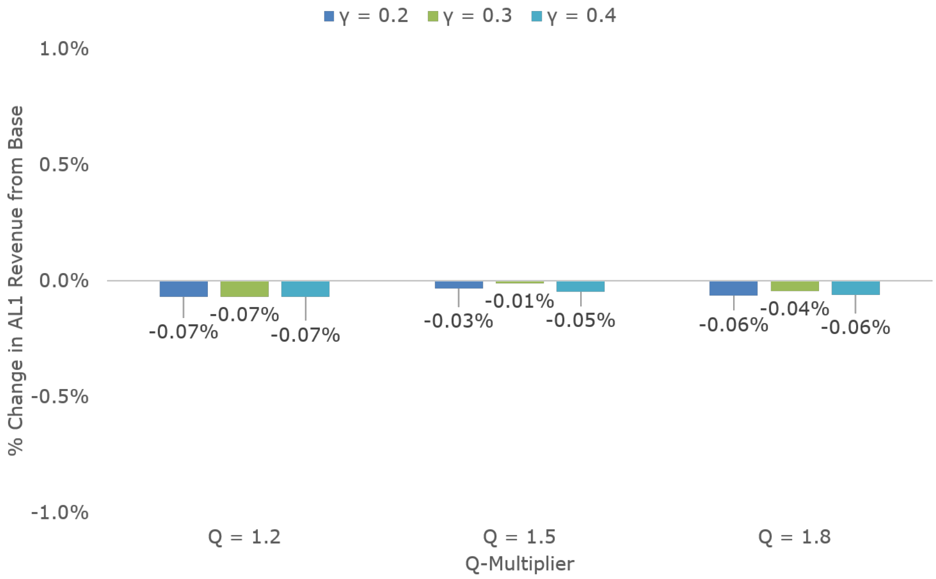

Figure 6: Percent change in AL1 revenue from base when AL1 use Leisure PDynA with a variety of input Q-multipliers and coefficients of variation γ

From Figure 6, it would be easy to conclude that Leisure PDynA has a negligible effect on passenger behavior. However, the revenue neutrality of the heuristic masks the underlying dynamics of Leisure PDynA on both business and leisure bookings, as shown in Figure 7. Leisure PDynA has a significant effect on revenues early in the booking process, when leisure (type $w = 2$) passengers are more likely to arrive. Giving these leisure passengers more favorable assortments with less expensive products leads to an increase in leisure revenues in early time periods. However, as it moves closer to departure, the airline comes to regret selling those seats to early-booking leisure passengers at low fares, as this action displaces late-arriving business passengers who are willing to pay higher fares.

Therefore, while Leisure PDynA increases leisure revenues early in the booking process, it also leads to a decline in business passenger revenues relative to the base later in the booking process. Overall, this reduction in business passenger revenues counters the leisure revenue gains, and results

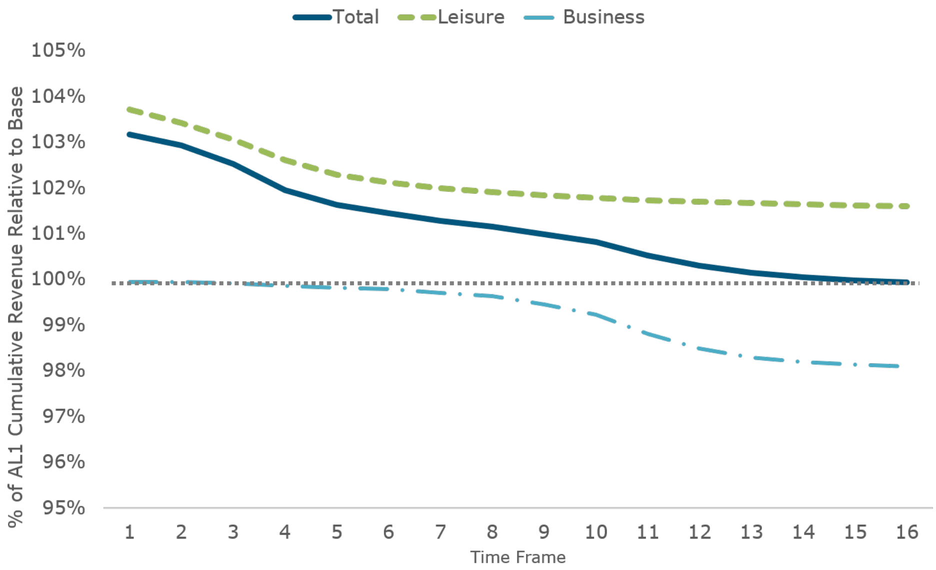

Figure 7: Percent of AL1 cumulative revenue relative to base by passenger type when AL1 uses Leisure PDynA (Input Q-multiplier = 1.2, $\gamma_{w,m} = 0.3$)

in a neutral net revenue performance for the heuristic. In a following section, we will see if we can improve on this heuristic by offering dynamic discounts to leisure passengers in situations when only relatively expensive itinerary products are made available in S^*.

4.3. Results: Probabilistic PFDynA (Customized dynamic pricing)

We now turn our attention to the PFDynA dynamic pricing heuristics. Recall that these heuristics worked by potentially changing the vector of prices offered to arriving passengers. Specifically, if k is the least-expensive itinerary product in S^*, we compute either an increment to fare f_k for business passengers of type $w = 1$ or a discount for business passengers of type $w = 2$, following decision rule (12).

4.3.1. Business PFDynA

The revenue impacts of Business PFDynA (dynamic pricing) as compared to Business PDynA (assortment optimization) are shown in Figure 8 for AL1. As the figure shows, the two heuristics perform similarly at most Q-multiplier estimates. For instance, with a Q-multiplier estimate of 2.0, customized dynamic pricing (Business PFDynA) results in a 0.54% increase in revenue for AL1, as compared to a 0.52% increase in revenue for AL1 when customized assortment optimization (Business PDynA) is used.

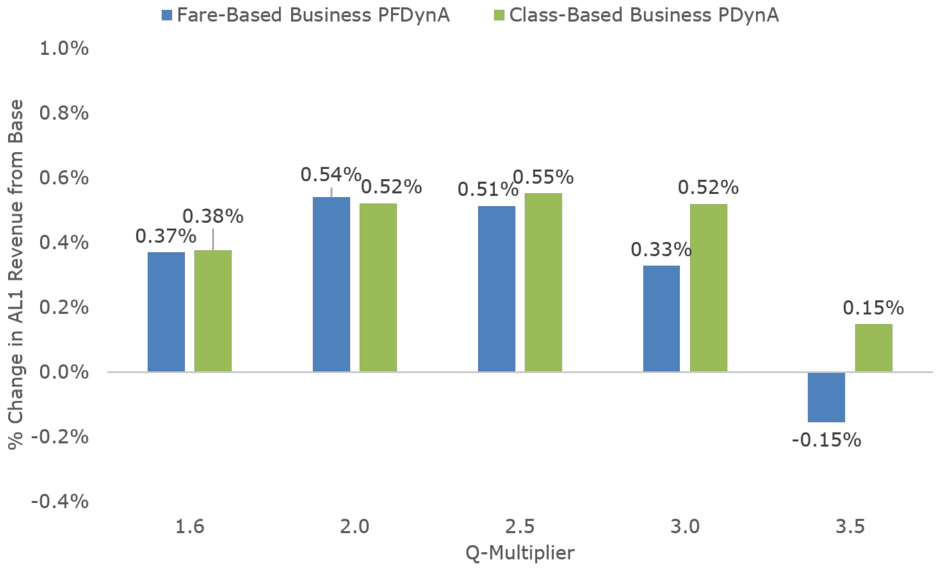

Figure 8: Percent change in AL1 revenue from base when AL1 uses Business PDynA or Business PFDynA (Various Q-Multipliers, $\gamma = 0.3$)

This performance is likely due to the bounds $[l, u]$ that are imposed on the fare increment Δ_k^*. Recall from Table 2 that the increment Δ_k^* is bounded by 0 (no increment) and the difference in fares between itinerary products $k-1$ and k. Therefore, in many cases, applying Business PFDynA is equivalent to selling itinerary product k at fare f_{k-1}, while Business PDynA would remove product k from the assortment S^*, leaving itinerary product $k-1$ as the least expensive option. In both cases, the lowest available price would be $f_{k-1} > f_k$.[9]

Since Business PFDynA performs so similarly to its customized assortment optimization counterpart, many of the observations from Section 4.2.1. also apply to Business PFDynA. For instance, Business PFDynA also increases passenger yield while reducing load factors, leading to a net increase in revenues. Furthermore, when all airlines practice Business PFDynA, revenue increases nearly double from the case in which only one airline practices the heuristic. As shown in Figure

[9]Note that without this bound on Business PFDynA, business passengers would always be charged an identical price equal to $\arg\max f \cdot \text{Prob}(\theta_i \geq f)$, regardless of the time remaining to booking. This is because our estimate of the passenger WTP budget distribution does not change according to the time remaining in the booking process.

9, the heuristic is also stable in competition as a Nash equilibrium; revenues of all airlines decrease if one airline chooses to deviate from the all-airline equilibrium.

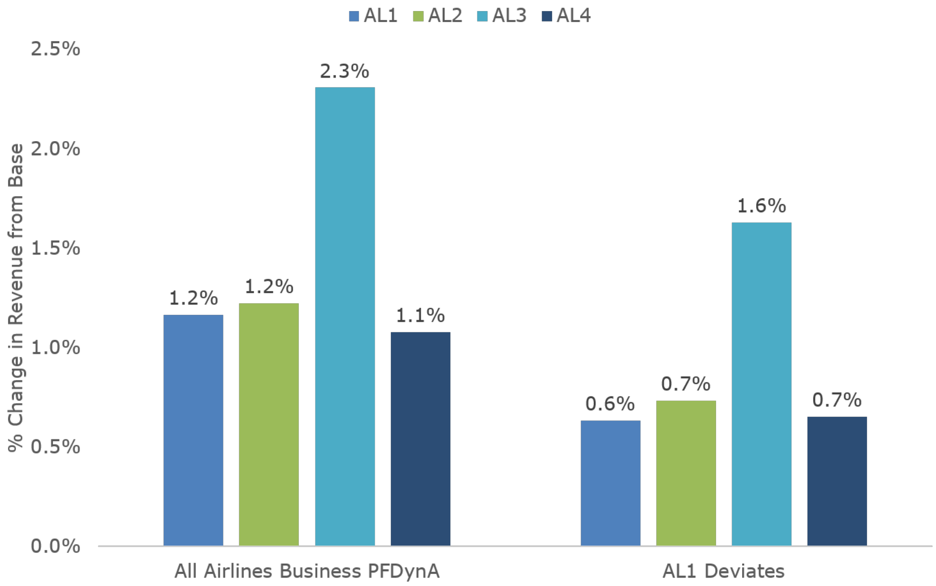

Figure 9: Percent change in revenue from base when various airlines use Business PFDynA (Input Q-multiplier = 2.0, $\gamma_{w,m} = 0.3$)

4.3.2. Leisure PFDynA

Recall that Leisure PFDynA works by giving discounts to type $w = 2$ passengers according to decision rule (12). The computed discount is bounded above by zero and below by $(f_{k+1} - f_k)$, where k is the least expensive itinerary product in assortment S^*. As opposed to Business PFDynA, which produced similar results to its customized assortment optimization counterpart, customized dynamic pricing in the form of Leisure PFDynA produces strikingly different results. While Leisure PDynA produced revenue neutral results, Leisure PFDynA leads to increases in revenue of between 2.1% and 4.6%, as shown in Figure 10.

However, caution is necessary when interpreting these high revenue impacts, particularly with lower input Q-multiplier values. For instance, with a Q-multiplier value of 1.2, the decision rule will always find it beneficial to give a discount to type $w = 2$ passengers, even when all products in the fare structure are included in assortment S^*. This behavior is shown in Figure 11; with a

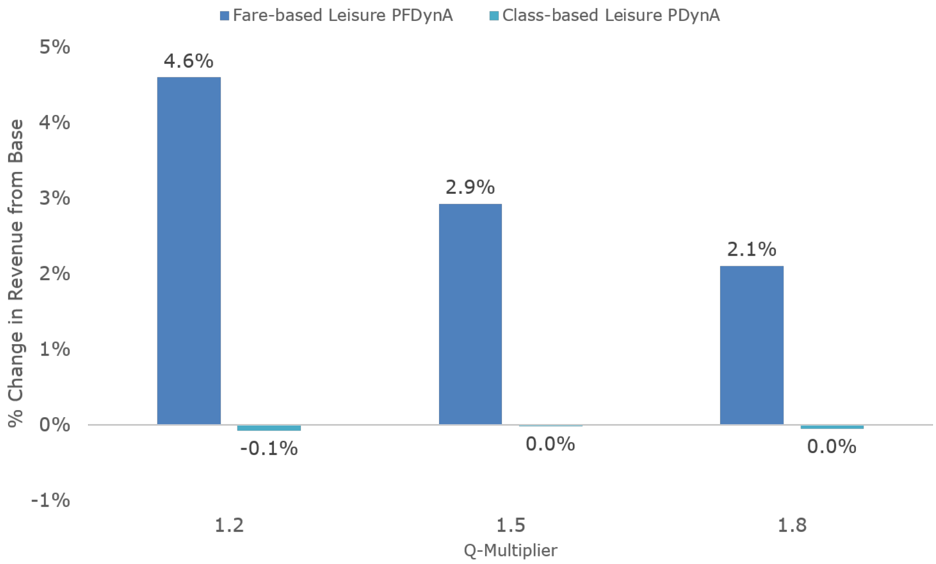

Figure 10: Percent change in AL1 revenue from base when AL1 uses Leisure PDynA or Leisure PFDynA (Various Q-Multipliers, $\gamma = 0.3$)

Q-multiplier of 1.2, 100% of Class 10 leisure passengers (the least expensive fare class in the fare structures in PODS Network U10) receive discounts. Class 10 passengers represent 51% of AL1's type $w = 2$ leisure passengers. As a result of receiving discounts in almost all situations, leisure passengers that would have ordinarily booked with other airlines flock to AL1 to take advantage of the lower fares.

But recall that in the generation of passenger WTP budgets in PODS, all passengers are assumed to be willing to purchase any itinerary at the lowest fare in the fare structure. In other words, all of the Class 10 passengers to which AL1 is giving a discount would have ordinarily booked at price f_{10} without a discount. While giving these passengers further discounts would increase any airline's revenues in isolation, this leaves the heuristic unstable in competition. As more airlines start providing discounts to low-fare-class leisure passengers, a "race to the bottom" ensues as airlines continue to undercut each other without stimulating new demand. As a result, if all airlines practice Leisure PFDynA with an input Q-multiplier of 1.2, each airline loses between 2.4% and 3.1% of its revenue relative to the base, as shown in Figure 12.

However, there is an easy way to fix this issue—eliminating discounts for itinerary product k

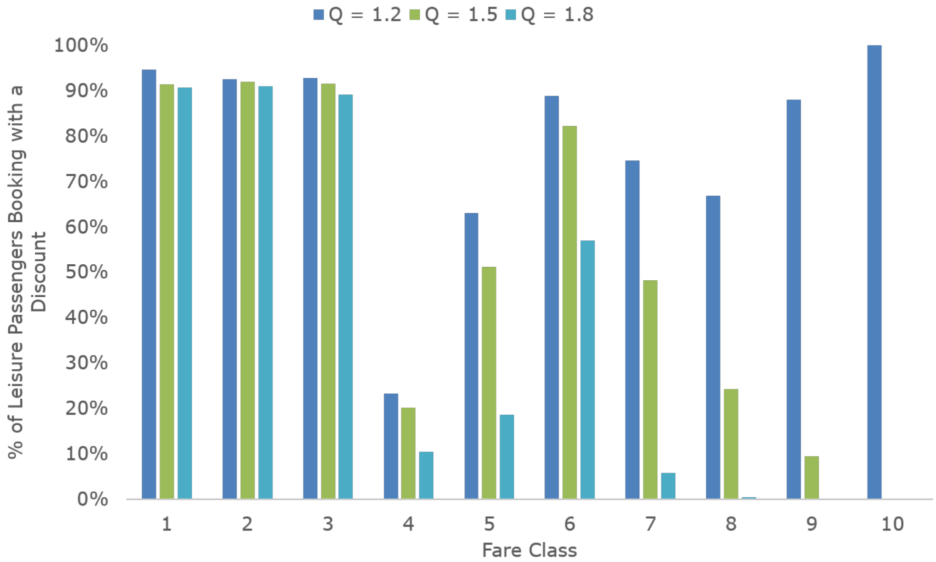

Figure 11: Percent of AL1 leisure passengers by fare class that book with a Leisure PFDynA discount when AL1 practices Leisure PFDynA (Various Q-multipliers, $\gamma = 0.3$)

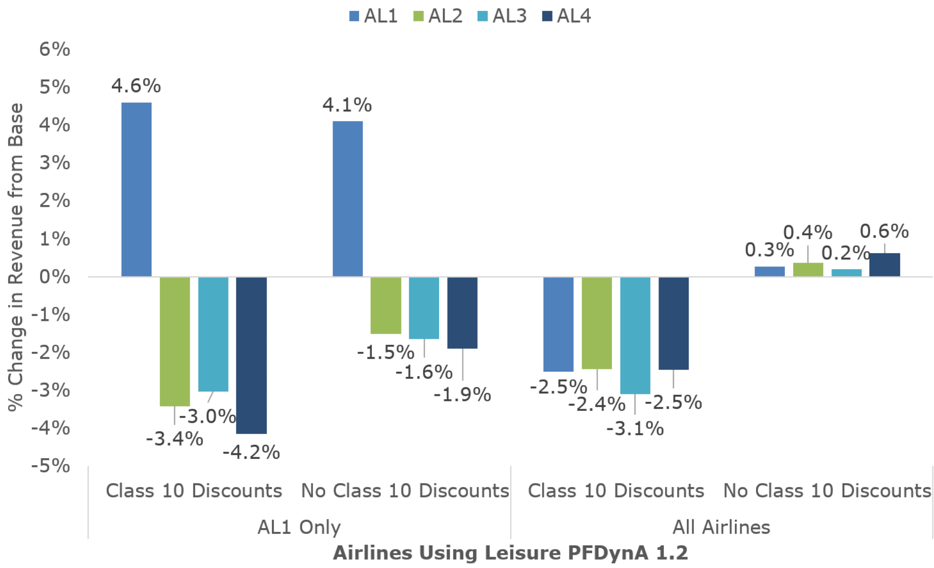

Figure 12: Percent change in revenue from base when airlines use Leisure PFDynA (Q = 1.2, γ = 0.3) with or without Class 10 discounts

when $k = n$ (the number of products in the fare structure). In other words, we no longer provide a discount for Class 10 passengers who would have ordinarily booked in that class without a discount. Figure 12 shows the effects of this action—eliminating Class 10 discounts leads to a slight reduction in AL1's revenue gains when it alone practices Leisure PFDynA, lowering revenue gains from 4.6% to 4.1%. And, this action also makes the heuristic stable in competition, as shown on the right side of Figure 12. When all airlines practice Leisure PFDynA (Q = 1.2) with Class 10 discounts, each airline loses a significant amount of revenue. However, when Class 10 discounts are eliminated, practicing the heuristic leads to slight revenue gains for all airlines as new demand is stimulated in the market.

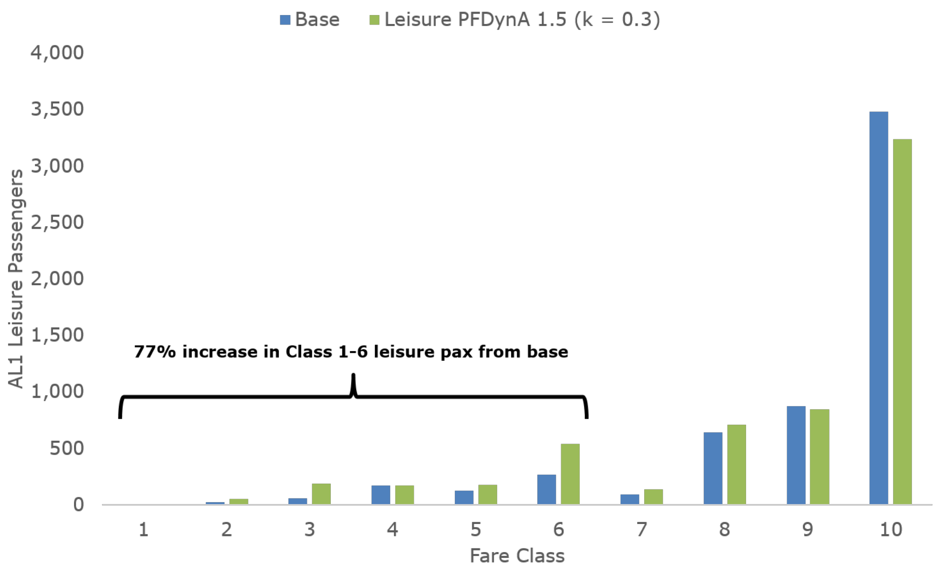

Figure 13: AL1 leisure passengers by fare class when AL1 uses Leisure PFDynA (Q = 1.5, $\gamma = 0.3$)

Despite giving discounts to passengers, Leisure PFDynA stimulates new demand in relatively higher fare classes. As shown in Figure 13, using Leisure PFDynA with a Q-multiplier of 1.5 leads to a 77% increase in Class 1–6 leisure passengers from the base. While the absolute number of passengers booking in these more expensive fare classes is small relative to the passengers booking in less-expensive fare classes, the heuristic leads to an increase in revenue of nearly 3%. Giving discounts to specific passengers when the itinerary products in the initial assortment S^* are relatively expensive leads to more passengers booking in relatively higher fare classes. As a result, Leisure PFDynA increases not only passenger yield, but also AL1's load factor. This makes Leisure PFDynA appear to be a Pareto-improving outcome; leisure passengers see lower fares, which leads to more leisure bookings, leading to higher revenues, load factors, and yields for the airline.

5. Conclusions and Future Work

In this paper, we proposed a categorial definition for customized airline revenue management in terms of assortment optimization and dynamic pricing. We then formulated a customized RM model in which, upon the observation of a passenger's type and an estimate of the passenger's WTP, the airline can make a customized adjustment to the assortment S^* that would ordinarily be offered with traditional airline RM. By adjusting either the products in the offered assortment or the prices of those products, the heuristics aim to increase revenues by increasing yield from high-WTP business passengers or generating new bookings from lower-WTP leisure passengers.

We then introduced several simple heuristics for either adjusting the products in the assortment, which we referred to as *class-based Probabilistic DynA (PDynA)*, or changing the prices of products in the assortment, which we called *fare-based Probabilistic DynA (PFDynA)*. We simulated these heuristics in the PODS revenue management simulator in a four-airline competitive international network. We found that both heuristics generally increased airline revenues by up to 4% over the base case, and that practicing the heuristics represented a stable Nash equilibrium in a competitive airline environment.

As the methods for customized airline revenue management continue to evolve, there are numerous ways in which the models in this paper could continue to be enhanced. For instance, the PDynA and PFDynA heuristics that were tested in PODS relied on a variety of assumptions regarding price-oriented passenger behavior in a single carrier market. More complex and nuanced passenger choice models could be used to estimate $P_w(k; f_k|S, R)$; more robust models would likely improve heuristic performance.

Incorporating competitor availability directly into the PDynA and PFDynA models could also potentially improve the performance of the heuristics. For example, it probably does not make sense for an airline to offer a price increment when its fare is already more expensive than a competitor's fare for a comparable itinerary product. On the other hand, if an airline offers the only nonstop flight in a market, it may be more likely to want to increment prices (Fiig et al., 2010). This is another component of the "observable state of nature" upon which customized airline RM bases its decision rules.

Finally, it is worth considering whether incorporating the customized assortment optimization and dynamic pricing mechanisms into the RM optimizer would yield benefits. This would in effect combine Stages 1, 3, and 4 of the customized RM schematic proposed in Section 3, and would be similar to the model proposed in Gallego et al. (2016). It is likely that in early stages of implementation of customized airline RM, airlines will be unwilling to significantly adjust the underlying models that drive the creation of the original assortment S^*. At the same time, customized RM as proposed in this paper can be seen as "overriding" the outputs of a traditional RM system, and it is worth considering whether informing the optimizer of customized offers would result in solutions that better approach an idealized optimum.

Such a development would come closer to the world of "true dynamic pricing" described by Westermann (2013), where "booking classes [are] rendered obsolete." It is also the world envisioned by supporters of IATA's New Distribution Capability, where customized offers are tailored to each specific passenger making a request. This approach, which fuses together assortment optimization and dynamic pricing, would represent a total rethinking of the ways airlines practice revenue management today, and significant effort would need to be put into resolving the IT/distribution and RM science challenges that such a system would create. Customized assortment optimization and dynamic pricing, as presented in this paper, can be seen as a first step towards this new frontier.

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