Modeling Flight Delay Propagation: A New Analytical-Econometric Approach

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Abstract: Flight delay and its propagation present a widespread phenomenon in the air transportation system, costing billions of dollars every year. To understand the delay propagation patterns and associated mitigation measures, this study proposes a novel analytical-econometric approach. Considering that airlines deliberately insert buffer into flight schedules and ground turnaround operations, an analytical model is developed to quantify propagated and newly formed delays that occur to each sequence of flights that an aircraft flies in a day, from three perspectives on the ways that delays are absorbed by the buffer. With delays computed from the analytical model, we further develop a joint discrete-continuous econometric model and use the Heckman’s two-step procedure to reveal the effects of various influencing factors on the initiation and progression of propagated delays. Results from the econometric analysis provide estimates on how much propagated delay will be generated out of each minute of newly formed delay, for the US domestic aviation system as well as for major airports and airlines. The impacts of other factors on the initiation and progression of propagated delay are also quantified. These results may help aviation system planners gain additional insights into flight delay propagation patterns and consequently prioritize resources while improving system overall performance. Airlines can also be better informed to assign buffer to their flight schedules to mitigate total propagated delay.

Keywords: Propagated delay, Newly formed delay, Flight buffer, Ground buffer, Analytical model, joint discrete-continuous econometric model
1 Introduction

Flight delay is a major challenge facing the air transportation system today. In the US, total flight delay cost is estimated to be over $30 billion each year (Ball et al., 2010). The cost comes from various sources, including additional use of crew, fuel, and aircraft maintenance; increase in passenger travel time; greater environmental externalities; and the macroeconomic impact of flight delay on other economic sectors. While solutions such as improvement in air traffic management (e.g., Ball et al., 2007; Swaroop et al., 2012) and aviation infrastructure investment (Zou and Hansen, 2012a; Zou, 2012) are expected to substantially reduce flight delays, an important means from the airlines’ perspective is to judiciously schedule flights. Specifically, by inserting additional times than the minimum necessary in and between flights, unexpected delays can be absorbed and consequently their propagation to downstream flights can be mitigated or avoided. The objective of this study is to advance the understanding of the delay propagation patterns and of the ways delays are mitigated by the additional times.

Propagated delay occurs because of connected resources involved in an initially delayed flight and flights downstream. The connected resources can be the aircraft, crew, passengers and airport resources. For example, the same aircraft flies multiple flight legs in a day. Delay of an earlier flight can sustain in the subsequent flights of the same aircraft. Flight crew can also switch between multiple aircraft, causing the delay from one flight to propagate across multiple flights. Connecting passengers at hub airports, like crew members, are also often responsible for the propagation of delay when a connecting flight has to wait for passengers from their previous delayed legs. Overall, delay can grow over the course of a day, with a small initial delay leading to larger delays later in the day. To mitigate this propagation effect, additional times are inserted in both flight schedules and ground turnaround operations. In this study, we term these additional times as flight buffer and ground buffer. The buffer can, therefore, be interpreted as the ex-ante amount of time built into a scheduled activity based on an expected amount of time for an activity plus an amount to maintain a level of on-time service. In this sense it is analogous to an inventory problem.

In the literature, analytical research on flight delay propagation dates back to 1998, when Beatty et al. (1998) used flight schedules of American airlines to calculate delay propagation multipliers. A delay propagation multiplier is a value which when multiplied with the initial delay yields the sum of all potential downstream delays plus the initial delay. Beatty et al. constructed delay trees by considering three causes for delay propagation: aircraft equipment, cockpit crew, and flight attendants. A delay tree can contain up to 50-75 flights for a flight early in the day that is connected to the rest of the system. However, the authors did not show how delays can be absorbed by flight and ground buffers. AhmadBeygi et al. (2008) also constructed tree structure to investigate how delay can propagate throughout an airline’s network in a day, for two US airlines. In the delay tree, the root delay, which occurs to the earliest flight in the tree, propagates to immediate downstream flights of the same aircraft. In cases that the cockpit crew and aircraft do not stay together, delay propagates to the flight the cockpit crew heads. Ground buffer was accounted for as a means to mitigate delay propagation. However, propagated delays due to connecting flight attendants or passengers and the delay recovery options were not included in the analysis. Welman et al. (2010) calculated delay multipliers for 51 US airports and estimated the reduction in total system delay if airport capacity was expanded. Buffer was not included while calculating propagated delay. Churchill et al. (2010) developed another analytical model, which explicitly accounts for ground buffers but not flight buffer.

Apart from the analytical approaches, other methods were used in flight delay propagation research. Wong and Tsai (2012) considered simultaneously flight and ground buffers and statistically estimated a survival model for flight delay propagation. Xu et al. (2008) used multivariate adaptive regression splines and found on average 5.3 minutes of generated delay and 2.1 minutes of absorbed delay across 34 Operational Evolution Plan (OEP) airports in the US. Pyrgiotis et al. (2013) developed a queuing engine to model flight delay formed at an airport. The formed delay was then utilized to modify the flight schedules and update demand at the airports. The two processes iterate to obtain the final local delay formed at an airport and its propagation.
While inserting buffer is an effective means to mitigate delay propagation, doing so has adverse effects. First, buffer makes flight schedules and ground turnaround times longer than the minimum necessary, reducing the utilization of aircraft and incurring greater capital cost. Second, because payment to airline crew is determined in part by the length of flight schedules, inserting flight buffer increases crew expenses. Third, with flight buffer, flights often arrive earlier when there is no delay. The landed aircraft are likely to encounter gate unavailability and thus have to wait in queue on the ramp with engines on, which increases airline operating cost and ramp congestion, and potentially aggravates passengers onboard (Hao and Hansen, 2014). These adverse effects prompt airlines to consider the best tradeoff when setting buffers in their flight schedules. Following this line of thought, AhmadBeygi et al. (2010) investigated the issue of reallocating ground buffer by re-timing flight departures, such that delay propagation can be mitigated without incurring additional planned cost. Monte Carlo simulations were performed in Schellenkens (2011) to establish the relationship between the duration of primary delays and the number of affected downstream flights. Focusing on delay propagation, Arikan et al. (2013) proposed several metrics to investigate the robustness of US airline schedules, and showed different airline strategies in setting flight and ground buffers.

Despite these efforts for modeling propagated flight delay, two important methodological gaps exist. First, no studies have so far looked into simultaneous use of flight and ground buffers in absorbing both propagated and newly formed delays. At a given flight arrival or departure point (or node, as we define in the next section), newly formed delay is defined as the delay that occurs during the immediate upstream operation (which can be either a flight or a ground turnaround); whereas propagated delay is delay that is rooted further upstream. Among the aforementioned studies, Churchill et al. (2010) and Arikan et al. (2013) considered only ground buffer. Although both flight and ground buffers were modeled in Schellenkens (2011), the authors did not differentiate between newly formed and propagated delays, nor the way the two types of delays are absorbed by buffer. How newly formed and propagated delays are absorbed by buffer was not discussed. In general, with the coexistence of propagated and newly formed delays, buffer could be used to first absorb newly formed delay and then propagated delay, or vice versa, or both at the same time. To fill this gap, the present study explicitly investigates three scenarios in which propagated and newly formed delays are absorbed by flight and ground buffers.

The second gap in the literature is a lack of statistical evidence of under what conditions newly formed delay will propagate downstream and, provided that it occurs, how the delay propagation is shaped by subsequent ground and flight buffers and the macro-congestion environment. Such statistical evidence is important to better understanding delay propagation patterns and mitigation strategies and build schedules more robust to the delays. In view of this gap, the present study makes a second contribution by proposing a joint discrete-continuous econometric model for the initiation and progression of propagated flight delay. The model consists of a latent variable based discrete choice model for propagated delay initiation and a regression model for the progression of propagated delay. Because flights that initiates propagated delays are not randomly picked from the population of flights, our estimation explicitly accounts for potential bias due to sample selection. Both a sequential and a simultaneous methods are used to obtain consistent model estimates. The contribution of initial delays, delay mitigating measures, and macro-congestion environment to delay propagation is statistically quantified, to our knowledge the first time in the literature. The estimates further allow us to infer the vulnerability to delay propagation at individual airports and for different airlines. We also discuss how these estimates can be useful to aviation system planners and airlines.

The remainder of the paper is organized as follow. Section 2 offers formal (although still conceptual) definitions for propagated and newly formed delays and the way delays buffer absorbs them. Section 3 describes the analytical model for computing the two types of delays under three scenarios, which reflect the different ways delays are absorbed by buffer. An application of the analytical model to the US domestic air transportation system is subsequently presented. In section 4, we present a joint discrete-continuous model to empirically characterize the initiation and progression of propagated delays, including model specification, estimation methods, discussion of results and their implications. Summary of findings is offered in Section 5.
2 Newly formed and propagated delays

Before quantifying delay and buffer, it is necessary to characterize the air transportation network. In this study we consider an air transportation network as a graph consisting of nodes and links. The node-link representation of air transportation networks is not new in the literature. Guimerà et al. (2005) and Barrat et al. (2004) used graphs to characterize air transportation networks in which nodes are airports and links are direct flights between the nodes. Our definition of nodes and links is somewhat different: each node corresponds to either an actual gate push-back or an actual gate arrival of a flight. Thus a node is associated with an airport. A link connects two neighboring nodes (time-wise). A link can be either a flight or a ground turnaround. To illustrate, Figure 1 shows the nodes and links of an aircraft’s itinerary in a time-space diagram. The aircraft flies three flights. Consequently the total number of nodes is six, one for each takeoff or landing. The time next to each node is the actual departure/arrival time (in standard time (e.g., Eastern Standard Time)) at the node, and the numbers next to the arrows indicate the flight delays at the nodes.

In this study, newly formed delay at a node refers to delay that occurs between the node and its immediate upstream node. For example, in Figure 1 the departure delay at DEN is 20 minutes. Since it is the beginning of the aircraft’s operation, all 20 minutes are newly formed delay at the DEN node. Between the DEN and DFW nodes (i.e., while the aircraft is flying), some newly formed delay must occur, as the arrival delay at DFW increases to 25 minutes. The newly formed delay will be counted at the DFW node. Newly formed delay can be caused by airport/airspace capacity constraints, aircraft breakdown and maintenance, airport security, etc. In contrast, propagated delay at a node refers to the part of delay that is rooted in some newly formed delay which occurs to the same aircraft but further upstream. Note that in principle delay can propagate between flight segments operated by the same aircraft, but can also propagate from one aircraft to another. One situation in the latter case is that delay of an aircraft during airport turnaround may spill over to an adjacent aircraft at the airport even if the aircraft arrived at the airport on time, due to shared use of airport resources (e.g. personnel to handle turnarounds). Because the focus of this paper is on aircraft-specific delay propagation, such inter-aircraft propagated delay will be considered as newly formed delay for the adjacent aircraft.

Figure 1: Illustration of link-node representation of aircraft itineraries

In the US domestic air transportation system, actual flight departure/arrival times and delays are all reported to the Federal Aviation Administration (FAA) and made publically available on the Bureau of Transportation Statistics (BTS) website. On the other hand, the distinction between newly formed and propagated delays is not readily made, due to at least two reasons. First, it is not publically known the exact
amount of buffer inserted in a flight schedule or a ground turnaround. In this paper we consider that buffer can be anticipated and is the difference between the scheduled flight (ground turnaround) time and the nominal time. Though not stochastic, buffer is heterogeneous across flight segments, airlines, aircraft, and seasons. Second, it is uncertain how buffer inserted on a link is used to absorb newly formed delay and propagated delay. As mentioned in Section 1, it can be that buffer first absorbs propagated delay and then newly formed delay, or vice versa, or absorbs both at the same time.

Given this uncertainty, we consider below three scenarios for delay absorption. In the first scenario, buffer is used to first absorb propagated delay. Any remaining buffer will be used to reduce newly formed delay. In the second scenario, the absorption order is reversed. In the third scenario, no priority is given while absorbing the two types of delays, i.e., buffer reduces both delays simultaneously.

3 The analytical model

3.1 Computing flight and ground buffers

The analytical model for computing propagated and newly formed delays requires identifying the amount of buffer inserted in flight schedules and ground turnaround operations. In line with the uncertainty of aircraft operations and nominal operation time being a function of flight segment, airline, aircraft, and seasonal characteristics, we consider alternative values for flight and ground buffers. Specifically, we use $5^{th}$, $10^{th}$, and $20^{th}$ percentile values of the actual gate-to-gate travel times among flights with departure delay as the nominal operation time, for each combination of flight segment, airline, aircraft category, and season.¹ We consider only flights with departure delay, as these flights clearly have more incentive to fly as efficiently in order to catch up with the schedule. Not choosing the absolute minimum actual time makes the calculation more robust to measurement error, and reduces the influence of unusually favorable conditions, such as strong tailwinds (Zou and Hansen, 2012b). Similarly, we use $25^{th}$, $50^{th}$, and $75^{th}$ percentile values of actual ground turnaround times among flights with arrival delay as the nominal operation time for ground turnaround, for each combination of airline, aircraft category, and season. Considering different percentile values allows us to gauge the robustness and sensitivity of the results.

Once the nominal operation times for the flight and ground turnaround are obtained, we calculate flight and ground buffers as follows. Consider that the trajectory of an aircraft in a day cover nodes $i = 1, 2, ..., I$. $I$ must be an even number as each flight involves a departure node and an arrival node. Let $U_{i-1,i}^F, \forall i = 2, 4, ..., I$ and $U_{i-1,i}^G, \forall i = 3, 5, ..., I - 1$ denote the nominal operation times for the flight and ground turnaround for the respective link $(i, i - 1)$. Consequently the buffer on a flight link is:

$$B_{i-1,i} = \max\{0, (t_i^F - t_{i-1}^F) - U_{i-1,i}^F\}, \forall i = 2, 4, ..., I \tag{1}$$

The buffer on a ground turnaround link is:

$$B_{i-1,i} = \max\{0, (t_i^G - t_{i-1}^G) - U_{i-1,i}^G\}, \forall i = 3, 5, ..., I - 1 \tag{2}$$

where $B_{i-1,i}$ is the buffer on link $(i - 1, i)$; $t_i^F$ is the scheduled departure (or arrival) time of the aircraft at node $i$. Note that the buffers in (1) and (2) are truncated below zero as we consider buffer to be zero if the scheduled time is less than the nominal operation time. In this case, we consider that the flying (or turnaround) has a lower nominal operation time, which is equal to the scheduled time.

3.2 Computing propagated and newly formed delays

With flight and ground buffers calculated, now we proceed to computing propagated and newly formed delays. Recall that three scenarios may be possible as to how buffer is used to absorb the two types of delays.

¹ In this study we group aircraft into two categories: wide body and narrow body jets. Operations by regional jets and turboprops are not observed for the airlines studied (in fact, such aircraft are operated by the subsidiaries of the airlines studied. See for example, Zou et al. (2014)).
Below we detail the methodologies for the three scenarios. Although the description is based on flight links, the methodologies apply in a similar way to computing delays on ground links.

We use a simple time-space graph to represent a flight in Figure 2. The horizontal line denotes time and the vertical line represents space. The flight is scheduled to depart from node $i - 1$ at $t_{i-1}^s$ and arrive at node $i$ at $t_i^s$. The amount of flight buffer on link $(i - 1, i)$ is $B_{i-1,i}$. If there were no departure delay at node $i - 1$ and flying from $i - 1$ to $i$ is also free from any en-route delay, then the arrival time of the flight at node $i$ would be earlier than $t_i^s$ by $B_{i-1,i}$. In Figure 2, however, the flight departs late by $O_{i-1}$ (note that $O_{i-1}$ can be a mix of newly formed delay at node $i - 1$ and propagated delay from the upstream of node $i - 1$). With this departure delay, the best arrival time at node $i$ would be $bt_i^a$ if the aircraft flies nominal time, with an amount of $B_{i-1,i}$ departure delay absorbed by flight buffer on link $(i - 1, i)$. The $t_i^n$ represents the normal arrival time at node $i$ given the initial departure delay, which is equal to actual departure time plus scheduled flight length. If a newly formed delay further occurs while the aircraft flies from $i - 1$ to $i$, a late arrival such as $t_i^a$ in Figure 2 would result. The observed delay at each node is the difference between scheduled arrival (departure) and actual arrival (departure) time. Decomposition of this observed delay into newly formed and propagated delay is discussed based on following three scenarios.

![Figure 2: Time-space graph of a single flight](image)

**Scenario 1:**

Scenario 1 postulates that buffer first absorbs newly formed delay. After newly formed delay is entirely eliminated, any remaining buffer absorbs propagated delay. Referring to Figure 2, if the observed delay at node $i$ is greater than at node $i - 1$, i.e., $O_i > O_{i-1}$, it means that some newly formed delay is added to node $i$ even after buffer $B_{i-1,i}$ is used. Because newly formed delay has higher priority over propagated delay for being absorbed, the propagated delay to node $i$ remains $O_{i-1}$. If $O_i \leq O_{i-1}$, then there is no newly formed delay at node $i$. Part of the propagated delay from node $i - 1$, equaling $O_{i-1} - O_i$, is absorbed by flight buffer $B_{i-1,i}$. We explicate the two cases in Appendix A. Below we present the unified algorithm to calculate the newly formed delay and propagated delay in a recursive manner from the first node of an aircraft in a day. At the first node, the observed delay $O_1$ is all attributed to newly formed delay $N_1$.

**Algorithm 1: Computing propagated and newly formed delays under Scenario 1**

1. At the first node of an aircraft $(i = 1)$, $N_1 = O_1$. There is no propagated delay.

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2 In this study, we consider observed delays that are truncated below zero, i.e., earlier than scheduled departures (arrivals) will be given zero observed delay values.
2. For each subsequent node \( i = 2, \ldots, I \),

2.1. Compute propagated delay whose root is the immediate upstream node \( i - 1 \): \( p_{i-1,i} = N_{i-1} \times \min(1, \frac{o_i}{o_{i-1}}) \);

2.2. Compute propagated delays whose roots are further upstream nodes \( k = 1, \ldots, i - 2 \): \( p_{k,i-1} = p_{k,i-1} \times \min(1, \frac{o_i}{o_{i-1}}) \);

2.3. Compute newly formed delay at node \( i \): \( N_i = \sum_{k=1}^{i-1} p_{k,i} \).

Note that an implicit proportional assumption is made in steps 2.1-2.2. Specifically, buffer used to absorb propagated delays from upstream at node \( i \) will be allocated proportionately to \( N_{i-1} \) and \( p_{k,i-1} \) (\( k = 1, \ldots, i - 2 \)) according to their original amounts. We also apply this assumption in scenarios 2 and 3. Our argument is that—absent empirical evidence—it is unreasonable to justify that buffer prefers to absorb any specific part of upstream propagated delays. An “equal” allocation seems more plausible to consider the way that buffer absorbs upstream propagated delays. Nonetheless, it may still be interesting to explore alternative ways to allocate buffer, although this is beyond the scope of the current work.

**Scenario 2:**

Scenario 2 postulates that buffer first absorbs propagated delay. After propagated delay is entirely eliminated, any remaining buffer absorbs newly formed delay. As propagated delay in general has roots at multiple upstream nodes, we again assume that propagated delays rooted in different upstream nodes are absorbed while maintaining their proportions. Newly formed delay at a node is the difference between observed delay and propagated delay. Different from scenario 1, buffer needs to be explicitly involved in delay computation. Depending on the relative length of observed delays between two consecutive nodes and the length of buffer, four cases are possible. We explicate the four cases in Appendix B. Below we present the unified algorithm for the computation of propagated and newly formed delays under the four cases.

**Algorithm 2: Computing propagated and newly formed delays under Scenario 2**

1. At the first node of an aircraft \( (i = 1) \), \( N_1 = O_1 \). There is no propagated delay.

2. For each subsequent node \( i = 2, \ldots, I \),

2.1. Compute \( B_{i-1,i}^{a} = \max(B_{i-1,i}, O_{i-1} - O_i) \); (see Appendix B for discussion of \( B_{i-1,i}^{a} \))

2.2. Compute propagated delay from the immediate upstream node \( i - 1 \): \( p_{i-1,i} = N_{i-1} \times \left[ 1 - \min(1, \frac{B_{i-1,i}^{a}}{O_{i-1}}) \right] \);

2.3. Compute propagated delay from further upstream nodes \( k = 1, \ldots, i - 2 \): \( p_{k,i} = p_{k,i-1} \times \left[ 1 - \min(1, \frac{B_{i-1,i}^{a}}{O_{i-1}}) \right] \);

2.4. Compute newly formed delay at node \( i \): \( N_i = O_i - \sum_{k=1}^{i-1} p_{k,i} \).

Similar to Algorithm 1, the min operators above allow for differentiating between cases 2(a)/(c) and 2(b)/(d).

**Scenario 3:**

Scenario 3 postulates that buffer simultaneously reduces newly formed delay and propagated delay in proportion to the share of the two types of delays. The conceptual difficulty here is that the amounts of propagated delay and newly formed delay at a node before buffer absorption are not known a priori. However, proportionate absorption suggests that the ratio of propagated to newly formed delays remains...
constant before and after delay absorption. We let $x$ be the amount of buffer on link $(i - 1, i)$ that is used to absorb propagated delay from upstream to node $i$. Then $B_{i-1,i} - x$ is the amount of buffer on link $(i - 1, i)$ that absorbs newly formed delay at node $i$. The constant ratio requires that:

$$\frac{o_{i-1}}{o_i - (o_{i-1} - x)} = \frac{o_{i-1} - x}{o_i - (o_{i-1} - x)}$$

(3)

The left hand side (LHS) and right hand side (RHS) of Equation (3) represent the ratios of propagated delay to newly formed delay at node $i$, without and with the use of buffer respectively. On the LHS, the numerator is the propagated delay (if there were no buffer) to node $i$, which equals the observed delay at node $i - 1$. In the denominator, $O_i - x$ is the amount of propagated delay after using buffer. Since $O_i$ is the total observed delay, $O_i - (O_{i-1} - x)$ is the amount of newly formed delay with buffer. $O_i - (O_{i-1} - x)$ needs to be added by $B_{i-1,i} - x$, which is the amount of newly formed delay that is absorbed by buffer, to obtain the total newly formed delay absent buffer on link $(i - 1, i)$. The LHS should be equal to the ratio of propagated delay and newly formed delay with buffer, i.e., $O_i - (O_{i-1} - x)$. Solving (3) yields

$$x = \frac{B_{i-1,i}}{B_{i-1,i} + O_i} O_i$$

(4)

Therefore, propagated delay at node $i$ is

$$P_i = O_{i-1} - x = O_{i-1} \frac{O_i}{B_{i-1,i} + O_i}$$

(5)

Substituting $P_i$ by $\sum_{k=1}^{i-1} p_{k,i}$ and $O_{i-1}$ by $N_{i-1} + \sum_{k=1}^{i-2} p_{k,i-1}$ gives

$$\sum_{k=1}^{i-1} p_{k,i} = (N_{i-1} + \sum_{k=1}^{i-2} p_{k,i-1}) \frac{O_i}{B_{i-1,i} + O_i}$$

(6)

As in scenario 1 and 2, we assume that propagated delays rooted in all upstream nodes are absorbed while maintaining their proportions. Then we can decompose propagated delay as follows:

$$p_{i-1,i} = N_{i-1} \frac{O_i}{B_{i-1,i} + O_i}$$

(7)

$$p_{k,i} = p_{k,i-1} \frac{O_i}{B_{i-1,i} + O_i}, \quad \forall k = 1 ... i - 2$$

(8)

Newly formed delay at node $i$ is the difference between observed delay and propagated delay:

$$N_i = O_i - P_i = O_i - O_{i-1} \frac{O_i}{B_{i-1,i} + O_i} = O_i \frac{B_{i-1,i} + O_i - O_{i-1}}{B_{i-1,i} + O_i}$$

(9)

Equation (9) holds true for any values of $O_{i-1}, O_i$ and $B_{i-1,i}$, except when $B_{i-1,i} + O_i < O_{i-1}$. This is similar to case 2 (c) (see Appendix B). If this occurs, we adjust the buffer value to $O_{i-1} - O_i$. The implicit assumption by doing this adjustment is that no new delay is formed on link $(i - 1, i)$ (as $N_i = 0$ from (9)). Consequently all observed delay at node $i$ (i.e., $O_i$) is propagated delay.

The above computation of propagated and newly formed delays is summarized by the following algorithm.

**Algorithm 3: Computing propagated and newly formed delays under Scenario 3**

1. At the first node of an aircraft ($i = 1$), $N_1 = O_1$. There is no propagated delay.
2. For each subsequent node $i = 2, ..., I$,
   1. Compute $B_{i-1,i}^a = \max(B_{i-1,i}, O_{i-1} - O_i)$;
   2. Compute propagated delay from the immediate upstream node $i - 1$: $p_{i-1,i} = N_{i-1} \frac{O_i}{B_{i-1,i}^a + O_i}$;
2.3. Compute propagated delay from further upstream nodes \( k = 1, \ldots, i - 2 \): \( p_{k,i} = p_{k,i-1} * o_i / b_{i-1,i} + o_i \).

2.4. Compute newly formed delay at node \( i \): \( N_i = O_i - \sum_{k=1}^{i-1} p_{k,i} \).

So far our discussion has been focused on identifying delays that are propagated to a given node from all upstream nodes. With \( p_{k,i} \)'s (\( \forall k < i \)) computed, we can also compute total propagated delay (TPD) rooted in node \( k \) to all its downstream nodes, as follows:

\[
TPD_k = \sum_{i=k+1}^{I} p_{k,i} \quad \forall k = 1, 2, \ldots I - 1
\]

(10)

\( TPD_k \) will be the main variable of interests in Section 4.

This concludes the methodologies for computing propagated and newly formed delays at any given node in an air transportation network. Next, we apply the methodologies using historic flight data in the US to obtain insights about delay propagation and formation patterns.

3.3 Data

We use US domestic flight data from eight major carriers (listed in Table 1) in the first quarter of 2007, when the system had one of the worse flight on-time performance records in history. Detailed activity information for each flight is collected from the BTS Airline On-Time Performance database (BTS, 2013a). Based on aircraft model information from the BTS Schedule B-43 inventory database (BTS, 2013b), we group aircraft into two categories: narrow body and wide body. The aircraft model/category information is then merged into the flight operation dataset by matching aircraft tail numbers.

Data filtering is needed to eliminate erroneous recording, such as: (i) teleportation, i.e. an aircraft landed at one airport, but the next departure was from a different airport; and (ii) a flight's departure time was earlier than the arrival time of its previous flight leg. We further remove cancelled and diverted flights, and the associated flights of the same aircraft of the same day. All observations on the transition days between standard and daylight saving times are excluded as these records can be subject to inconsistent recording. After the filtering and conversion process, our final dataset consists of 642,227 observations, covering flights flying between 168 US airports. The total numbers of aircraft from the eight carriers are 2513. Table 1 summarizes the number of flights by airline and aircraft category.

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Narrow Body</th>
<th>Aircraft Categories</th>
<th>Total</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>Wide Body</td>
<td></td>
</tr>
<tr>
<td>American</td>
<td>58,038</td>
<td>4,250</td>
<td>62,288</td>
</tr>
<tr>
<td>Alaska</td>
<td>28,975</td>
<td>0</td>
<td>28,975</td>
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<tr>
<td>Continental</td>
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<td>1,265</td>
<td>72,494</td>
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<tr>
<td>Delta</td>
<td>87,381</td>
<td>8,714</td>
<td>96,095</td>
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<tr>
<td>Northwest</td>
<td>84,693</td>
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<tr>
<td>United</td>
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<td>106,565</td>
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<td>US Airways</td>
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<tr>
<td>Southwest</td>
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<tr>
<td>Total</td>
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</table>

Using the merged dataset, we then compute nominal operation times for ground turnaround and flight, ground and flight buffers, and propagated and newly formed delays for each flight node. The next subsection presents the computation results.
3.4 Results

3.4.1 Nominal operation times for ground turnaround and flight

We compute nominal turnaround time on ground links by airline and aircraft category (as the data is only for one quarter, seasonal variation is not relevant). Table 2 shows the average nominal ground turnaround times based on different percentile values. As expected, the nominal ground turnaround times for narrow body aircraft is significantly lower than for wide body aircraft. For Alaska, Northwest, and Southwest, no wide body operations are recorded. Among the narrow body fleets, the lowest nominal ground turnaround times are observed in Southwest, with 22 minutes at the 25th percentile. This is not surprising, as Southwest operates predominantly point-to-point services which do not require as much time as for a hub-and-spoke carrier for passenger connection at hub airports. The situation for wide body aircraft is more complicated. We observe that, when 50th and 75th percentile values are used, the calculated nominal ground turnaround time for Continental will be 108 and 277 minutes, much larger than for the other airlines. This may be because many Continental wide body flights do not fly much in a day and spend most time on the ground, resulting in inter-flight ground time much larger than nominal turnaround time. In contrast, the calculated nominal ground turnaround times are more consistent across airlines when 25th percentile values are considered. In view of this, we only use 25th percentile to compute nominal ground turnaround time in the rest of the paper.

<table>
<thead>
<tr>
<th>Carrier</th>
<th>Aircraft category</th>
<th>25th Percentile</th>
<th>50th Percentile</th>
<th>75th Percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>American</td>
<td>NB</td>
<td>38</td>
<td>45</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>WB</td>
<td>61</td>
<td>71</td>
<td>87</td>
</tr>
<tr>
<td>Alaska</td>
<td>NB</td>
<td>34</td>
<td>43</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>NB</td>
<td>42</td>
<td>52</td>
<td>70</td>
</tr>
<tr>
<td>Continental</td>
<td>WB</td>
<td>62</td>
<td>108</td>
<td>277</td>
</tr>
<tr>
<td></td>
<td>NB</td>
<td>40</td>
<td>48</td>
<td>62</td>
</tr>
<tr>
<td></td>
<td>WB</td>
<td>54</td>
<td>63</td>
<td>87</td>
</tr>
<tr>
<td>Northwest</td>
<td>NB</td>
<td>38</td>
<td>50</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>WB</td>
<td>37</td>
<td>45</td>
<td>59</td>
</tr>
<tr>
<td>United</td>
<td>NB</td>
<td>39</td>
<td>48</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>WB</td>
<td>56</td>
<td>69</td>
<td>91</td>
</tr>
<tr>
<td>US airways</td>
<td>WB</td>
<td>22</td>
<td>26</td>
<td>31</td>
</tr>
</tbody>
</table>

Note: NB: Narrow Body, WB: Wide Body

Figure 3 (a) presents the nominal flight operation times based on the 5th percentile observed flight times, for each combination of flight segment-airline-aircraft category. The flight times are colored and binned into 10 intervals. In total, 3583 unique nominal flight operation times are obtained. As an illustration of further detailed results, Figures 3 (b) and 3 (c) plot nominal flight operation times based on the 5th percentile actual flight times for American’s narrow body and wide body aircraft operations. It can be seen that narrow body flights cover more and generally shorter-haul flight segments than wide body flights. The colors show that nominal flight operation times highly correlate with flying distance, as is expected.
Figure 3 (a): Nominal flight operation times based on the 5th percentile actual flight times

Figure 3 (b): Nominal flight operation times based on the 5th percentile actual flight times for American’s narrow body flights
3.4.2 Ground and flight buffers

Table 3 presents summary statistics for ground and flight buffers based on different percentile values from the actual times. The average ground buffer is around 23 minutes based on the 25th percentile actual turnaround time. The minimum value is 0 and the values are widely dispersed with standard deviation of 74 and a maximum value of 1015 minutes. Possible reasons for the wide dispersion may be due to scheduled aircraft maintenance between two flights, or simply that airlines do not schedule certain aircraft to fly much in a day (thus spending most time on the ground). In contrast to ground buffer, flight buffer is smaller and less dispersed. This implies greater heterogeneity associated with ground operations than flight operations.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground buffer based on the 25th percentile</td>
<td>23.30</td>
<td>73.46</td>
<td>0</td>
<td>1015</td>
</tr>
<tr>
<td>Flight buffer based on the 5th percentile</td>
<td>18.05</td>
<td>8.32</td>
<td>0</td>
<td>80</td>
</tr>
<tr>
<td>Flight buffer based on the 10th percentile</td>
<td>15.10</td>
<td>7.56</td>
<td>0</td>
<td>74</td>
</tr>
<tr>
<td>Flight buffer based on the 20th percentile</td>
<td>9.80</td>
<td>6.30</td>
<td>0</td>
<td>63</td>
</tr>
</tbody>
</table>

3.4.3 Propagated delay

Table 4 shows summary statistics for total propagated delay (TPD) rooted in each node, with nominal operation times based on the 5th percentile actual ground turnaround time and the 25th percentile actual flight time. Note that many nodes have zero newly formed delays, thus having no TPDs. In Table 4, only nodes with non-zero TPDs are considered in generating the summary statistics. TPDs for departure and arrival nodes are reported separately, as we speculate that departure and arrival nodes may exhibit different delay propagation patterns (this is further discussed in section 4). The average TPD ranges from 19.48 to 44.96 minutes. We note that some propagated delays are too large, which may be caused by unexpected aircraft maintenance and measurement errors. To reduce the influence of such outliers which may stem to our further analysis, nodes with TPDs larger than the 75th percentile value plus 1.5 times the inter-quantile range are dropped from the dataset. Recall that by definition a node is associated with a flight. Flights that are linked to the dropped flight by the same aircraft are also dropped. Depending on the scenarios, this excludes about 3-5% of flights from the original dataset. Summary statistics for the remaining data are shown in the lower half of Table 4.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4: Summary statistics for total propagated delays (TPDs)
<table>
<thead>
<tr>
<th>Before dropping outlying values</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total propagated delay from departure nodes</td>
<td>44.96</td>
<td>99.15</td>
<td>8e-16</td>
<td>2736.2</td>
</tr>
<tr>
<td>Total propagated delay from arrival nodes</td>
<td>26.83</td>
<td>49.46</td>
<td>3e-16</td>
<td>1776.49</td>
</tr>
<tr>
<td><strong>Scenario 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total propagated delay from departure nodes</td>
<td>43.30</td>
<td>93.40</td>
<td>2e-10</td>
<td>2693.25</td>
</tr>
<tr>
<td>Total propagated delay from arrival nodes</td>
<td>26.22</td>
<td>43.48</td>
<td>1e-09</td>
<td>1682.7</td>
</tr>
<tr>
<td><strong>Scenario 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total propagated delay from departure nodes</td>
<td>28.43</td>
<td>74.85</td>
<td>1e-15</td>
<td>2694.23</td>
</tr>
<tr>
<td>Total propagated delay from arrival nodes</td>
<td>19.48</td>
<td>36.62</td>
<td>1e-10</td>
<td>1630.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>After dropping outlying values</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Scenario 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total propagated delay from departure nodes</td>
<td>19.08</td>
<td>23.49</td>
<td>8e-16</td>
<td>108.18</td>
</tr>
<tr>
<td>Total propagated delay from arrival nodes</td>
<td>14.63</td>
<td>16.27</td>
<td>3e-16</td>
<td>74.43</td>
</tr>
<tr>
<td><strong>Scenario 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total propagated delay from departure nodes</td>
<td>19.04</td>
<td>22.07</td>
<td>2e-10</td>
<td>99.16</td>
</tr>
<tr>
<td>Total propagated delay from arrival nodes</td>
<td>13.86</td>
<td>14.52</td>
<td>2e-09</td>
<td>69.16</td>
</tr>
<tr>
<td><strong>Scenario 3</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total propagated delay from departure nodes</td>
<td>8.84</td>
<td>11.74</td>
<td>1e-15</td>
<td>55.85</td>
</tr>
<tr>
<td>Total propagated delay from arrival nodes</td>
<td>8.58</td>
<td>10.42</td>
<td>1e-10</td>
<td>51.94</td>
</tr>
</tbody>
</table>

### 3.4.4 Temporal distribution of propagated and newly formed delays

In this subsection we present system total propagated and newly formed delays associated with the nodes aggregated by hour over all days in the dataset. The aggregation uses local time for each node. Again, departure and arrival nodes are considered separately. The first column in Figure 4 (i.e., (a), (c), (e)) represents newly formed delay at flight departure nodes (in dark blue) and total propagated delay from these nodes (in light blue). The second column in Figure 4 (i.e., (b), (d), (f)) shows newly formed delay at flight arrival nodes and total propagated delay from these nodes, using the same color notation. In each plot, x-axis denotes time and y-axis indicates total delay minutes. The first row (i.e., Figures 4(a) and 3(b)) corresponds to scenario 1; the second row scenario 2; and the third row scenario 3.

Overall, both newly formed and propagated delays are considerably lower from after midnight till early morning. This is the period in a day when most aircraft are grounded. Newly formed delay under scenario 1 is smaller than under scenarios 2 and 3, which is not surprising as in scenario 1 buffer first absorbs newly formed delay. In contrast, propagated delay under scenario 1 seems consistently larger than under the other two scenarios, which is especially true at flight departure nodes. Newly formed departure delays are generally greater than newly formed arrival delays, which suggest that departure delay is a more prominent phenomenon than arrival delay in the system.
3.4.5 Spatial distribution of propagated and newly formed delays

The significance of propagated and newly formed delays varies by location as well as by time of a day. As an illustration, Figure 5 presents total newly formed departure delays, aggregated by airport, in six time intervals: 7 am - 8 am, 10 am - 11 am, 1 pm - 2 pm, 4 pm - 5 pm, 7 pm - 8 pm, and 10 pm - 11 pm EST. The associated total propagated delays are also shown. The computation of newly formed and propagated delays is based on scenario 1. The airports cover all that appear in our dataset.

Overall, strong spatial and temporal heterogeneity exists for both types of delays. Delays peak during 7 am - 8 am and 7 pm - 8 pm, local time. Hub airports such as Chicago O’Hare experience high delays.
throughout a day, whereas smaller airports have more prominent time-of-day delay variations. In addition, a lag in operation and delay exists between the eastern and western parts of the country. At 7 am - 8 am EST, airports on the east coast and in the mid-west already have considerable newly formed delays. These delays later generate considerable total propagated arrival delays downstream. Delays during the same hour are much less at western airports, since the local time is 4 am - 5 am and there is not so much traffic. At 10 am - 11 am, airports in the western region begin to show increased delays. At most airports, delays decrease after the morning peak. Then, starting around 4 pm - 5 pm EST, eastern airports start seeing a surge in evening traffic. At 10 pm - 11 pm EST, the eastern airports approach the end of an operation day with diminishing delays, whereas airports in the western part still have considerable newly formed (and consequently propagated) delays.
The competition between $N_i$ and buffer is absorbed together with $N_i$ for buffer.

The ground and flight buffers will further compete with propagated delay to node $i$, i.e., $N_i$, because $N_i$ will be absorbed with at least the same priority as $N_i$ under the two scenarios by available buffer. Under scenarios 2 and 3, $N_i$ will further compete with propagated delay to node $i$ (i.e., $\sum_{k=1}^{i-1} p_{k,i}$) for buffer.\textsuperscript{4} The initiation problem can be modeled as a discrete, binary choice problem, with a 0-1 dependent variable indicating whether a node generates non-zero TPD.

When modeling the progression of TPD, we are only concerned with nodes that have positive delays. These nodes comprise a non-random sample in the original dataset. We hypothesize that the progression of propagated delay is shaped by four groups of factors: i) the severity of the newly formed delay ($N_i$); ii) propagated delay to node $i$ ($\sum_{k=1}^{i-1} p_{k,i}$), which is absorbed together with $N_i$ downstream of node $i$; iii) downstream mitigating factors including ground and flight buffers; and iv) the macro-congestion environment downstream. It should be noted that simply applying Ordinary Least Squares (OLS) to regress $TPD_i$ on these factors would yield biased results due to non-random sample selection – the nodes selected to run regression are those which are more vulnerable to delay propagation than those unselected.

\textsuperscript{3} When appropriate, $B_{i+1}$ needs to be updated by $B_{i+1}$ as discussed in Algorithms 2 and 3 in subsection 3.2.

\textsuperscript{4} Under scenario 1, the competition between $N_i$ and $\sum_{k=1}^{i-1} p_{k,i}$ for buffer would be secondary compared to the competition between $N_i$ and $N_{i+1}$.
To prevent the potential bias brought by sample selection, we propose a joint discrete-continuous model that estimates initiation and progression of propagated delay simultaneously. A key in the joint model is that the error terms in the discrete delay initiation model and the continuous delay progression model are correlated. Below we detail the model specification.

### 4.1 Model specification

In our econometric model, the unit of observation is a departure or an arrival node defined in section 2. For the initiation of propagated delay, we introduce a 0-1 indicator variable $z_i$ for each node $i$ and an associated latent variable $z_i^*$ whose value is a linear combination of independent variables $w_i$ (with $\mathbf{y}$ being the combination coefficients) and a random error term $u_i$:

$$z_i^* = w_i \mathbf{y} + u_i$$

where as mentioned above two common variables in $w_i$ across all three scenarios are $N_i$ and $B_{i,i+1}$. Under scenarios 1 and 3, $N_{i+1}$ is further included in $w_i$; so is $\sum_{k=1}^{i-1} p_{k,i}$ under scenarios 2 and 3.

- **Scenario 1:** $N_i, B_{i,i+1}, N_{i+1}$;
- **Scenario 2:** $N_i, B_{i,i+1}, \sum_{k=1}^{i-1} p_{k,i}$;
- **Scenario 3:** $N_i, B_{i,i+1}, \sum_{k=1}^{i-1} p_{k,i}, N_{i+1}$;

The relationship between $z_i$ and the latent variable $z_i^*$ is that

$$z_i = 1 \text{ if } z_i^* > 0; \quad z_i = 0 \text{ if } z_i^* < 0$$

Assuming that $u_i$ follows a standard normal distribution, we model the sample selection (i.e., selecting nodes that have non-zero TPDs) using a Probit specification:

$$\operatorname{Prob}(z_i = 1| w_i) = \Phi(w_i \mathbf{y}); \quad \operatorname{Prob}(z_i = 0| w_i) = 1 - \Phi(w_i \mathbf{y})$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution.

The progression of delay propagation is modeled as a linear combination of independent variables $x_i$ (with $\mathbf{b}$ being the combination coefficients) and a random error term $\epsilon_i$:

$$y_i = x_i \mathbf{b} + \epsilon_i \text{ observed only if } z_i = 1$$

where $y_i$ is the total propagated delay from node $i$ (TPD$_i$). We consider two types of TPD$_i$ based on whether $i$ is a departure node (thus propagated delay from a newly formed departure delay) or an arrival node (thus propagated delay from a newly formed arrival delay). The hypothesis is that newly formed departure and arrival delays have different propagation patterns.

For the independent variables $x_i$, three groups of variables are included: delays at node $i$, ground and flight buffers on downstream links, and the macro-congestion environment downstream. The first group contains two variables: $N_i$ and $\sum_{k=1}^{i-1} p_{k,i}$. For $N_i$, we hypothesize that the severity of the newly formed delay positively correlates with the amount of delay it can propagate. Knowing TPD$_i > 0$, newly formed delay at node $i$ ($N_i$) will compete with propagated delay to node $i$ ($\sum_{k=1}^{i-1} p_{k,i}$) for being absorbed by buffer as both propagate downstream. Thus these two variables both affect the size of TPD$_i$.

The second group of variables, ground and flight buffers on downstream links, plays the role of reducing propagated delays. Recall that flight operations considered in this study are between 6 am and 10 pm. For the aircraft that is associated with the node of concern, we group and sum ground and flight buffers on downstream links of the node to each of the four time groups: 6 am – 10 am, 10 am – 2 pm, 2 pm – 6 pm, and 6 pm – 10 pm (buffers prior to the node of concern are given value zero). The grouping of downstream ground buffers is based on the local time of the departure node on the corresponding ground link. Similar
grouping and summation are performed on flight buffers, except that the grouping of flight buffers is based on the local time of the arrival node on a corresponding flight link.

The third group of variables, which reflects the macro-congestion environment downstream, controls for the propensity of forming new delays downstream of node $i$, which affects the extent to which $N_i$ further propagates or is absorbed. We introduce average airport arrival delay during each hour between 6 am and 10 pm local time (thus in total 16 such variables are constructed). The average airport arrival delay for a given hour is calculated by dividing the total flight delay minutes by the total number of flights that actually arrived at an airport in that hour (measured in local time). The airport-hour correspondence is identified by the downstream arrival nodes. Let us refer back to Figure 1 to illustrate this. If one is interested in total propagation delay from the departure node (DEN, 10:10 am), then the average airport hourly arrival delays included would be at (DFW, 1-2 pm), (PHX, 3-4 pm), and (LAS, 4-5 pm), all in local time. For hours that do not have a downstream arrival node, the average airport arrival delays take value zero.

We assume that $\varepsilon_i$ follows a normal distribution: $\varepsilon_i \sim N(0, \sigma^2_\varepsilon)$. In addition, the error terms $u_i$ and $\varepsilon_i$ are correlated with correlation coefficient $\rho$. The intuition is that the sample of nodes with non-zero propagated delays would be nodes that are more vulnerable to delay propagation. The error terms are specified to have a joint normal structure:

$$
(u_i, \varepsilon_i) \sim N[(0)^T, \begin{pmatrix} 1 & \rho \\ \rho & \sigma^2_\varepsilon \end{pmatrix}]
$$

Using the results on multivariate distributions (Johnson and Kotz, 1974; Greene, 2012), the expected value for $y_i$ conditional on $z_i = 1$ is

$$
E(y_i|z_i = 1) = E(y_i|z_i^* > 0) = E(y_i|u_i > -w_i y) = x_i \beta + E(\varepsilon_i|u_i > -w_i y) = x_i \beta + \rho \sigma_\varepsilon \frac{\phi(w_i y)}{\Phi(w_i y)} = x_i \beta + \beta_\lambda \frac{\phi(w_i y)}{\Phi(w_i y)}
$$

where $\beta_\lambda = \rho \sigma_\varepsilon$ and $\phi(\cdot)$ is the probability density function (PDF) of the standard normal distribution. Thus

$$
y_i|z_i^* > 0 = E(y_i|z_i = 1) + \varepsilon_i = x_i \beta + \beta_\lambda \frac{\phi(w_i y)}{\Phi(w_i y)} + \varepsilon_i
$$

It is now clear that OLS regression of $y_i|z_i^* > 0$ on $x_i$ would produce inconsistent estimates for $\beta$ due to omitting variable $\frac{\phi(w_i y)}{\Phi(w_i y)}$.

### 4.2 Estimation methods

Two possible methods may be used to estimate the coefficients in the econometric model for the initiation and progression of propagated delays. One is Heckman’s two-step procedure (Heckman, 1979), which estimates the initiation and progression parts of the model sequentially. The other, full information maximum likelihood (FIML) method, estimates both parts simultaneously. We have experimented with both methods, which yield consistent results. To keep the length of the paper we only report results from the Heckman’s procedure, which is also computationally easier.

Under the Heckman’s two-step procedure, the initiation of propagated delays is estimated first. The results are then used in the second step to estimate the progression part. Specifically:

**Step 1:** Estimate Probit model (13) by maximum likelihood to obtain estimated coefficients $\tilde{\beta}$. For each observation with non-zero propagated delay, compute $\frac{\phi(w_i \tilde{\beta})}{\Phi(w_i \tilde{\beta})}$.

**Step 2:** Estimate the linear regression model (17) by OLS on the sample with non-zero propagated delays. This gives estimates $\hat{\beta}$ and $\hat{\beta}_\lambda$. 

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The caveat here is that the covariance matrix for \( \hat{\beta}_A \) differs from the traditional one under OLS. This is because the error term \( \epsilon_i | z_i = 1, w_i, x_i \) is heteroskedastic ( Heckman, 1979; Greene, 2012):

\[
\text{var}(\epsilon_i | z_i = 1, w_i, x_i) = \sigma^2(1 - \rho^2 \delta_i), \quad \text{where} \quad \delta_i = \frac{\phi(w_i \gamma)}{\Phi(w_i \gamma)} \left( \frac{\phi(w_i \gamma)}{\Phi(w_i \gamma)} + w_i \gamma \right) \cdot
\]

Using the estimated \( \hat{\delta}_i = \frac{\phi(w_i \hat{\gamma})}{\Phi(w_i \hat{\gamma})} \left( \frac{\phi(w_i \hat{\gamma})}{\Phi(w_i \hat{\gamma})} + w_i \hat{\gamma} \right) \), and residuals \( \hat{\epsilon}_i \), a consistent estimator of \( \sigma^2 \) can be obtained as

\[
\hat{\sigma}^2 = \frac{\hat{\epsilon}_i^2 + \hat{\beta}_1 \sum \delta_i}{N} \tag{18}
\]

where \( N \) is the number of observations used in Step 2 regression. The consequent estimator for the correlation coefficient \( \rho \) is

\[
\hat{\rho} = \frac{\hat{\beta}_1}{\hat{\sigma}_x}
\tag{19}
\]

Due to Greene (1981), the variance-covariance matrix for \( \hat{\beta}_A \) is

\[
\text{Var}[\hat{\beta}_A] = \hat{\sigma}_x^2 [X'X]^{-1} \left[ X' \left( I - \hat{\rho}^2 \hat{\Delta} \right) X + \hat{Q} \right] [X'X]^{-1}
\tag{20}
\]

where \( X_* = \begin{bmatrix} x_1 & \phi(w_2 \hat{\gamma}) & \cdots & x_i & \phi(w_i \hat{\gamma}) & \cdots & x_N & \phi(w_N \hat{\gamma}) \end{bmatrix} \); \( I - \hat{\rho}^2 \hat{\Delta} \) is a diagonal matrix of dimension \( N \times N \) with \( (1 - \hat{\rho}^2 \delta_i) \) being the \( i \)th \((i = 1, \ldots, N)\) diagonal element; \( Q = \hat{\rho}^2 [X_*' \hat{\Delta} W \text{Var}[\hat{\gamma}][W' \hat{\Delta} X_*] \) in which \( W = \begin{bmatrix} w_1; & \cdots; & w_i; & \cdots; & w_N \end{bmatrix} \) and \( \text{Var}[\hat{\gamma}] \) is the variance-covariance matrix for \( \hat{\gamma} \) obtained from Step 1.

### 4.3 Estimation results

As mentioned in subsection 4.1, separate models for \( TPD_i \) with \( i \) being a departure node and an arrival node are estimated. In conjunction with the three scenarios for delay absorption, in total six models are estimated for given percentile values that determine flight and ground buffers. In the interest of space, detailed estimation results of the models using 5th percentile value for flight buffer and 25th percentile value for ground buffer computation are presented in Appendix C. Later in this subsection, we also report the sensitivity of model estimates to other percentile choices.

Across all six models, most of the coefficients are significant and have expected signs. For the initiation of propagated delay, the coefficients of newly formed departure (arrival) delay at the current node \( i \) is positive, confirming a positive correlation between newly formed delay and the likelihood of delay propagation. The magnitude of influence is about 3-4 times larger at a departure node than at an arrival node. The negative coefficient for \( B_{i,i+1} \) is expected: greater buffer reduces the chance of propagated delay being initiated. The buffer effect is smaller than the effect of newly formed delay, which is especially true when the nodes chosen correspond to arrivals.

Under scenarios 1 and 3 where propagated delay has at most equal priority with newly formed delay for being absorbed by buffer, the extent that \( N_i \) will be absorbed depends on the amount of \( N_{i+1} \). The higher \( N_{i+1} \), the greater chance that \( N_i \) will propagate to node \( i + 1 \). This conjecture is corroborated by the positive coefficient for \( N_{i+1} \). However, the effect of \( N_{i+1} \) is considerably lower than the effect of \( N_i \). Under scenarios 2 and 3, the coefficients for \( \sum_{k=1}^{i-1} p_{k,i} \) are also positive, as expected. Because \( \sum_{k=1}^{i-1} p_{k,i} \) competes with \( N_i \) for buffer, the higher the \( \sum_{k=1}^{i-1} p_{k,i} \), the greater chance that \( N_i \) will propagate to node \( i + 1 \). Similar to the \( N_{i+1} \) variable, the effect of \( \sum_{k=1}^{i-1} p_{k,i} \) is smaller than the effect of \( N_i \).

Turning to the progression part for propagated delay, the estimates show that \( N_i \) significantly affects the size of \( TPD_i \). Again, the positive effect is larger at departure nodes than at arrival nodes (note that there is a negative coefficient for arrival nodes under scenario 2). \( \sum_{k=1}^{i-1} p_{k,i} \) has a positive coefficient across all progression models (except for delay at arrival nodes in Table A.3), which can be explained as follows: now that \( N_i \) is known to progress downstream, it competes with \( \sum_{k=1}^{i-1} p_{k,i} \) for absorption given an amount of buffer.
The ground and flight buffer variables generally have the expected negative signs, with the exception under scenario 1, where significant positive coefficients are obtained for departure nodes. The counter-intuitive sign may imply that scenario 1 be a less likely characterization of the way delays are absorbed by buffer. The general trend for the ground buffer coefficients is that the delay mitigation effect is greater at earlier time periods than later. As an example, ground buffer in time group 1 is 2.9 ((-0.047/(-0.016)) times more effective than in time group 4 for the model in the last column in Table C.3. Such a trend seems less prominent for flight buffers.

Finally, the coefficients for the average airport arrival delays in each hour are mostly positive, which supports our hypothesis that a more congested macro-environment downstream aggravates delay propagation. The effects do not strictly increase over the course of hours, but follows a more general first-increase-then-decrease trend, which is in accord with conventional wisdom: on the one hand, as it is getting into the middle of a day, air traffic and consequently congestion grow, making propagated delay more vulnerable to further progression; on the other hand, as the day is approaching its end, air traffic congestion dwindles. As a result the impact of macro-congestion environment on delay propagation diminishes.

Recall that the results presented here are based on the 5th percentile value for flight buffer and 25th percentile value for ground buffer computation. To test the sensitivity of the results to difference percentile choices, we also estimate the model with other buffer values. For the interest of space, here we only present in Table 5 the coefficient estimates for newly formed departure and arrival delays in the progression part of the model with different flight buffers. As buffer is the difference between the scheduled time and a chosen percentile time (given in parenthesis in table 5), a greater percentile value corresponds to a smaller buffer value. We observe that both coefficients for newly formed departure and arrival delays decrease with the increase in buffer values, with greater sensitivity occurring to arrival delay. The explanation for the decreasing trend is intuitive: if one believes that there is less buffer in the schedule, then the effects of newly formed delays on delay propagation would be perceived greater.

Table 5: Sensitivity of newly formed departure and arrival delay coefficients in the progression part of the model with different values for flight buffer

<table>
<thead>
<tr>
<th>Average calculated flight buffer</th>
<th>Coefficient of newly formed delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>18.05 mins (5th percentile)</td>
<td>Departure node</td>
</tr>
<tr>
<td></td>
<td>0.767</td>
</tr>
<tr>
<td>15.1 mins (10th percentile)</td>
<td>Arrival node</td>
</tr>
<tr>
<td></td>
<td>0.463</td>
</tr>
<tr>
<td>9.8 mins (20th percentile)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.801</td>
</tr>
<tr>
<td></td>
<td>0.551</td>
</tr>
</tbody>
</table>

4.4 Implication for airport- and airline-specific delay propagation

This subsection extends the previously estimated model by looking into airport- and airline-specific progression of propagate delays. The results presented here are based on scenario 3, although the general insights are consistent if considering the other two scenarios or with FIML for model estimation.

For airport-specific delay propagation, we re-estimate the model for 10 airports that had the most flight arrivals in the dataset. More specifically, given an airport, only observations whose departure (or arrival) node corresponds to the airport chosen are used for model estimation. Figure 6 (a) reports the estimated coefficients for newly formed departure and arrival delays. We find that six out of the 10 airports have larger coefficients than the system average for newly formed departure delay, and five airports have larger coefficients for newly formed arrival delay. The estimated coefficients for newly formed delays are important to airport authorities as it can also be interpreted as the average reduction in propagated delay as a result of newly formed delay reduction. Take DTW for example. If the newly formed arrival delay for all flights using the airport were reduced by 1 minutes due to airport capacity expansion, then the propagated delay of these flights would be 0.903 minutes less. Comparison of these airport-specific coefficients for newly formed delays may help aviation system planners such as the FAA gain deeper insights into flight
delay propagation patterns and consequently prioritize resources while improving system overall performance.

We similarly estimate airline-specific models for each of the eight airlines in our dataset. Figure 6 (b) plots the estimated coefficients for both newly formed departure and arrival delays. Southwest has the largest coefficient for both newly formed departure and arrival delays, which is consistent with the fact that the airline has the lowest buffer (see Table 2). As Southwest operates a predominately point-to-point model, the results also suggest a negative relationship between delay the propagation multiplier and the extent of point-to-point service structure adopted. Among the remaining network carriers, the coefficients for newly formed departure delay are fairly close. The same is true for the coefficients for newly formed arrival delay with the exception of American. These airline-specific coefficients, together with other coefficient estimates in the joint discrete-continuous model for the initiation and progression of propagated delay not shown here, can be useful to airlines in assigning buffer to flight schedules and controlling airline-wide operational performance to mitigate total propagated delay.

Figure 6: Delay propagation multiplier for different (a) airports and (b) airlines

5 Conclusion

A flight may be late arriving at or departing from an airport due to delays that occurred much earlier in the day. A small amount of initial flight delay can propagate downstream and compromise the on-time performance of the overall operations of the same aircraft. Ground and flight buffers are introduced so as to absorb both newly formed and propagated delays. To understand the delay propagation pattern and in particular the role played by different buffers in mitigating delay propagation, this study makes two methodological contributions: first, an analytical model is developed providing three different perspectives to quantify how flight delay propagates from any upstream node to any downstream node of the same aircraft. This analytical approach is applied to the US air transportation system using publically available data. Second, we develop a joint discrete-continuous econometric model and use Heckman’s two-step procedure to reveal the effects of various influencing factors on the initiation and progression of propagated flight delays. The results are generally robust to the different scenarios that delays are absorbed by buffers

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While constructing a metric to precisely measure the extent to which an airline uses point-to-point vs. hub-and-spoke services is beyond the scope of the present paper, previous empirical research did reveal that the extent of hubbing by Southwest is substantially lower than the other airlines considered here. For example, Baumgarten et al. (2014) obtained the average Hubbing Concentration Index value of Southwest to be 87 between 2003 and 2010. During the same period the index values for United, American, and Delta were all above 250. Similar significant gaps were confirmed in Martin and Voltes-Dorta (2009). We thank one of the anonymous reviewers for raising this point.
and to the buffer values chosen. Estimation of the joint econometric model allows us to gauge the importance of newly formed delay to delay propagation specific to individual airports and airlines.

The analytical and empirical models developed in this study may help decision making by airport system planners and airlines. With airport-specific insights into delay propagation, airports and the civil aviation authorities could be better informed about where to increase capacity while making infrastructure investment decisions. Airlines could potentially use the analytical model to analyze their flight scheduling practice and identify and improve critical flights and ground operations, where inserting additional buffer will be worthwhile. Given the still dearth of knowledge on flight delay propagation vis-à-vis its pervasiveness in the air transportation system, the ability to understand and mitigate propagated flight delays is of great relevance. Our study contributes to enhancing this ability.

The present approaches can be further extended in a few directions. Among them, we think the following three are of special interest. First, this study focuses on delay propagation within each aircraft. Delay propagation from one aircraft to a second aircraft, which occurs due to passenger and crew connections and due to connected airport resources, is not explicitly accounted for. In effect, in our paper such propagated delay is considered as newly formed delay for the latter aircraft. The difficulty is that existing data do not provide direct information to distinguish between this type of propagated delay and the newly formed delay that is truly caused by an aircraft itself. It would be interesting to investigate potential ways to make the distinction. Second, our dataset contains observations (although a very small portion) with extended periods of ground turnaround due to aircraft maintenance and mechanical checks, the latter required by the FAA. Such activities may be scheduled or may occur unexpectedly. When scheduled, such operations affect our analysis by giving a large buffer time for ground turnaround (as seen in Table 3). On the other hand, if they occur unexpectedly, they result in large newly formed ground delays. This large newly formed ground delay, in turn, results in very large amount of total propagated delay (as seen in Table 4). These errors, if not controlled, also propagates to the regression model which may bias the results obtained. Future research may look into ways to identify such activities and assign different operational turnaround times. Third, the finding that newly formed departure and arrival nodes have different delay propagation patterns may depend on the way we compute total propagated delay from a node, and nominal flight and ground turnaround times. Further investigation may be warranted to test whether this finding holds with alternative ways to construct these variables. Overall, we hope that our effort will stimulate further investigations along this line of research towards deeper knowledge on the phenomenon of flight delay propagation and means to mitigate it.

Appendix A

Case 1 (a): $O_i > O_{i-1}$

In this case, the propagated delay from upstream nodes to node $i$, $P_i$, would not be absorbed at all and be equal to the observed delay at node $i - 1$, $O_{i-1}$:

$$P_i = O_{i-1}$$  \hspace{1cm} (A.1)

Newly formed delay at node $i$, $N_i$, after using flight buffer $B_{i-1,i}$, is the difference between observed delay and propagated delay at the node:

$$N_i = O_i - P_i = O_i - O_{i-1}$$  \hspace{1cm} (A.2)

The propagated delay $P_i$ can be disentangled by root. In principle, the roots can be all upstream nodes from the first node to node $i - 1$. Letting $p_{k,i}$ denote the amount of propagated delay to node $i$ whose root is in upstream node $k = 1, \ldots, i - 1$, $P_i$ can be rewritten as

$$P_i = \sum_{k=1}^{i-1} p_{k,i}$$  \hspace{1cm} (A.3)
Given $O_i > O_{i-1}$, propagated delay from upstream nodes is not absorbed at all between nodes $i-1$ and $i$. Therefore we have
\[
\sum_{k=1}^{i-1} p_{k,i} = O_{i-1} = \sum_{k=1}^{i-2} p_{k,i-1} + N_{i-1}
\]
where $p_{k,i-1} = p_{k,i}$ for $k = 1, \ldots, i-2$. Consequently
\[
p_{i-1,i} = N_{i-1}
\]
which says that the propagated delay whose root is the immediate upstream node is simply the newly formed delay at the immediate upstream node.

**Case 1 (b): $O_i \leq O_{i-1}$**

In this case, flight buffer absorbs all newly formed delays that occur during the flight from node $i-1$ to node $i$. No newly formed delay is present at node $i$. In addition, an amount of $O_{i-1} - O_i$ propagated delay will be reduced, leaving only $O_i$ as the propagated delay at node $i$. Mathematically,
\[
N_i = 0
\]
\[
P_i = O_i - N_i = O_i
\]

Similar to case 1 (a), $P_i = \sum_{k=1}^{i-1} p_{k,i}$. To calculate each $p_{k,i}$, we assume that the propagated delays with different roots are absorbed while maintaining their proportions. Thus root-specific propagated delays at node $i$ are obtained by
\[
p_{i-1,i} = N_{i-1} * \frac{O_i}{O_{i-1}}
\]
\[
p_{k,i} = p_{k,i-1} * \frac{O_i}{O_{i-1}} \forall k = 1, \ldots, i-2
\]

**Appendix B**

**Case 2 (a): $O_i > O_{i-1}$ and $B_{i-1,i} < O_{i-1}$**

In this case, buffer on link $(i-1, i)$ is smaller than the observed delay at node $i-1$. All the buffer is used to absorb the propagated delay from all upstream nodes to node $i$. After absorption, an amount of propagated delay equal to $O_{i-1} - B_{i-1,i}$ remains at node $i$:
\[
P_i = O_{i-1} - B_{i-1,i}
\]
which is equivalent to
\[
P_i = O_{i-1} [1 - \frac{B_{i-1,i}}{O_{i-1}}]
\]

Substituting $P_i$ and $O_{i-1}$ by $\sum_{k=1}^{i-1} p_{k,i}$ and $N_{i-1} + \sum_{k=1}^{i-2} p_{k,i-1}$, we obtain
\[
\sum_{k=1}^{i-1} p_{k,i} = (N_{i-1} + \sum_{k=1}^{i-2} p_{k,i-1}) * [1 - \frac{B_{i-1,i}}{O_{i-1}}]
\]

Noting that propagated delays rooted in all upstream nodes are absorbed proportionally, we have
\[
p_{i-1,i} = N_{i-1} [1 - \frac{B_{i-1,i}}{O_{i-1}}]
\]
\[
p_{k,i} = p_{k,i-1} [1 - \frac{B_{i-1,i}}{O_{i-1}}] \quad \forall k = 1, \ldots, i-2
\]

Because $O_i > O_{i-1}$, the observed delay at node $i$ also contains some newly formed delay, which is
\[ N_i = O_i - P_i = O_i - O_{i-1} \left[ 1 - \frac{B_{i-1,i}}{O_{i-1}} \right] \]  
(A.15)

**Case 2 (b):** \( O_i > O_{i-1} \) and \( B_{i-1,i} > O_{i-1} \)

In this case, buffer not only fully absorbs all propagated delay from upstream nodes to node \( i \) (which is equal to observed delay at node \( i - 1 \)), but also reduces part of the newly formed delay. All delays in \( O_i \) are newly formed on link \((i - 1, i)\). Thus

\[ P_i = 0 \]  
(A.16)

\[ N_i = O_i - P_i = O_i \]  
(A.17)

**Case 2(c):** \( O_i < O_{i-1} \) and \( B_{i-1,i} < O_{i-1} \)

In this case, buffer is entirely used to absorb propagated delay. Still, as \( B_{i-1,i} < O_{i-1} \), some propagated delay equal to \( O_{i-1} - B_{i-1,i} \) remains at node \( i \). Newly formed delay at node \( i \) is the difference between the observed delay and the propagated delay, i.e., \( N_i = O_i - P_i = O_i - (O_{i-1} - B_{i-1,i}) = O_i + B_{i-1,i} - O_{i-1} \).

There is a caveat here. It is possible that \( O_i + B_{i-1,i} < O_{i-1} \). This occurs when buffer for this particular flight is actually larger than the calculated value \( B_{i-1,i} \). Recall that we consider \( 5^{th}/10^{th}/20^{th} \) percentile values of observed flight times as the nominal flight operation times. As a result, \( 5%/10%/20% \) of total flights would fly shorter than the nominal flight operation time. For these flights, our approach is to adjust their buffer values to \( O_i - O_{i-1} \). By doing so, we implicitly assume that zero new delay is formed. A unified expression encompassing this special circumstance is as follows:

\[ B_{i-1,i}^a = \max(B_{i-1,i}, O_{i-1} - O_i) \]  
(A.18)

\[ P_i = O_{i-1} - B_{i-1,i}^a = O_{i-1} \left[ 1 - \frac{B_{i-1,i}^a}{O_{i-1}} \right] \]  
(A.19)

\[ N_i = O_i - P_i = O_i + B_{i-1,i}^a - O_{i-1} \]  
(A.20)

The propagated delay in (A.19) can be similarly decomposed by root node as in (A.13) and (A.14).

**Case 2(d):** \( O_i < O_{i-1} \) and \( B_{i-1,i} > O_{i-1} \)

In this case, buffer on link \((i - 1, i)\) entirely eliminates propagated delay from upstream of node \( i \) (i.e., \( O_{i-1} \)). The observed delay at node \( i \), \( O_i \), is purely newly formed delay. Because \( B_{i-1,i} > O_{i-1} \), an amount of buffer equal to \( B_{i-1,i} - O_{i-1} \) will be used to absorbed newly formed delay. The expressions for \( P_i \) and \( N_i \) are exactly the same as in (A.16) and (A.17).

**Appendix C**

Table C.1: Model estimation results under scenario 1

<table>
<thead>
<tr>
<th>Scenario 1</th>
<th>Departure node</th>
<th>Arrival node</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initiation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Newly formed departure delay at node ( i )</td>
<td>0.244***</td>
<td>0.071***</td>
</tr>
<tr>
<td>Newly formed arrival delay at node ( i )</td>
<td>-0.020***</td>
<td>-0.0002***</td>
</tr>
<tr>
<td>Buffer on link ((i, i + 1)) ( B_{i(i+1)} )</td>
<td>-1.233***</td>
<td>-1.338***</td>
</tr>
<tr>
<td>Newly formed delay at node ( i + 1 ) ( N_{i+1} )</td>
<td>0.029***</td>
<td>0.021***</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.021***</td>
<td>-0.0002***</td>
</tr>
</tbody>
</table>

**Progression**

24
Newly formed departure delay at node $i$: $0.932^{***}$
Newly formed arrival delay at node $i$: $0.176^{***}$
Propagated delay to node $i$ ($\sum_{k=1}^{i-1} p_{k,i}$): $0.064^{***}$ $0.045^{***}$
Ground buffer in time group 1: $-0.058^{***}$ $-0.033^{***}$
Ground buffer in time group 2: $-0.042^{***}$ $-0.041^{***}$
Ground buffer in time group 3: $-0.006^{***}$ $-0.014^{***}$
Ground buffer in time group 4: $-0.003^{***}$ $-0.004^{***}$
Flight buffer in time group 1: $-0.032^{***}$ 0.003
Flight buffer in time group 2: $0.008$ -0.002
Flight buffer in time group 3: $0.034^{***}$ -0.008
Flight buffer in time group 4: $0.042^{***}$ -0.012^{***}
Delay_6: $0.135^{***}$ -0.083^{***}
Delay_7: $0.057^{***}$ 0.024^{*}
Delay_8: $0.076^{**}$ 0.043
Delay_9: $0.140^{***}$ 0.091^{***}
Delay_10: $0.067^{***}$ 0.037^{*}
Delay_11: $0.164^{***}$ 0.011
Delay_12: $0.224^{***}$ 0.108^{***}
Delay_13: $0.168^{***}$ 0.101^{***}
Delay_14: $0.156^{***}$ 0.134^{***}
Delay_15: $0.207^{***}$ 0.141^{***}
Delay_16: $0.209^{***}$ 0.112^{***}
Delay_17: $0.237^{***}$ 0.115^{***}
Delay_18: $0.224^{***}$ 0.088^{***}
Delay_19: $0.199^{***}$ 0.094^{***}
Delay_20: $0.141^{***}$ 0.073^{***}
Delay_21: $0.084^{***}$ 0.052^{***}
Constant: $4.645^{***}$ 36.634^{***}

Log-likelihood: 414,150 383,213
Observations: 414,150 383,213
$\rho$: -0.46 -1.00
$\sigma_e$: 14.52 22.29
$\beta_i$: -6.69 -22.29

*** significant at 1% level; ** significant at 5% level; * significant at 10% level.
Note: node $i$ refers to the node based on which the observation unit is identified.

<table>
<thead>
<tr>
<th>Scenario 2</th>
<th>Departure node</th>
<th>Arrival node</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Initiation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Newly formed departure delay at node $i$</td>
<td>$0.262^{***}$</td>
<td></td>
</tr>
<tr>
<td>Newly formed arrival delay at node $i$</td>
<td></td>
<td>$0.060^{***}$</td>
</tr>
<tr>
<td>Buffer on link $(i, i + 1)$ ($B_{i,i+1}$)</td>
<td>$-0.202^{***}$</td>
<td>$-0.023^{***}$</td>
</tr>
<tr>
<td>Propagated delay to node $i$ ($\sum_{k=1}^{i-1} p_{k,i}$)</td>
<td>$0.153^{***}$</td>
<td>$0.025^{***}$</td>
</tr>
<tr>
<td>Constant</td>
<td>$-1.022^{***}$</td>
<td>$-1.362^{***}$</td>
</tr>
</tbody>
</table>

**Progression**

Table C.2: Model estimation results under scenario 2
<table>
<thead>
<tr>
<th>Initiation</th>
<th>Departure node</th>
<th>Arrival node</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newly formed departure delay at node $i$</td>
<td>0.883***</td>
<td>-0.683***</td>
</tr>
<tr>
<td>Newly formed arrival delay at node $i$</td>
<td>-0.423***</td>
<td>-0.683***</td>
</tr>
<tr>
<td>Propagated delay to node $i$ ($\sum_{k=1}^{i-1} p_{k,i}$)</td>
<td>0.220***</td>
<td>-0.351***</td>
</tr>
<tr>
<td>Ground buffer in time group 1</td>
<td>-0.042***</td>
<td>-0.060***</td>
</tr>
<tr>
<td>Ground buffer in time group 2</td>
<td>-0.046***</td>
<td>-0.041***</td>
</tr>
<tr>
<td>Ground buffer in time group 3</td>
<td>-0.008***</td>
<td>-0.013***</td>
</tr>
<tr>
<td>Ground buffer in time group 4</td>
<td>-0.005***</td>
<td>-0.005</td>
</tr>
<tr>
<td>Flight buffer in time group 1</td>
<td>-0.145***</td>
<td>0.001</td>
</tr>
<tr>
<td>Flight buffer in time group 2</td>
<td>-0.189***</td>
<td>-0.036**</td>
</tr>
<tr>
<td>Flight buffer in time group 3</td>
<td>-0.076***</td>
<td>-0.013</td>
</tr>
<tr>
<td>Flight buffer in time group 4</td>
<td>-0.088***</td>
<td>-0.035***</td>
</tr>
<tr>
<td>Delay_6</td>
<td>0.034</td>
<td>0.072</td>
</tr>
<tr>
<td>Delay_7</td>
<td>0.429***</td>
<td>0.001</td>
</tr>
<tr>
<td>Delay_8</td>
<td>0.240***</td>
<td>-0.085</td>
</tr>
<tr>
<td>Delay_9</td>
<td>0.342***</td>
<td>-0.033</td>
</tr>
<tr>
<td>Delay_10</td>
<td>0.320***</td>
<td>0.065</td>
</tr>
<tr>
<td>Delay_11</td>
<td>0.340***</td>
<td>0.062</td>
</tr>
<tr>
<td>Delay_12</td>
<td>0.394***</td>
<td>0.076**</td>
</tr>
<tr>
<td>Delay_13</td>
<td>0.362***</td>
<td>0.092***</td>
</tr>
<tr>
<td>Delay_14</td>
<td>0.333***</td>
<td>0.072**</td>
</tr>
<tr>
<td>Delay_15</td>
<td>0.336***</td>
<td>0.100***</td>
</tr>
<tr>
<td>Delay_16</td>
<td>0.317***</td>
<td>0.062**</td>
</tr>
<tr>
<td>Delay_17</td>
<td>0.271***</td>
<td>0.055**</td>
</tr>
<tr>
<td>Delay_18</td>
<td>0.294***</td>
<td>0.067***</td>
</tr>
<tr>
<td>Delay_19</td>
<td>0.241***</td>
<td>0.072***</td>
</tr>
<tr>
<td>Delay_20</td>
<td>0.170***</td>
<td>0.056**</td>
</tr>
<tr>
<td>Delay_21</td>
<td>0.115***</td>
<td>0.047***</td>
</tr>
<tr>
<td>Constant</td>
<td>-8.557***</td>
<td>83.924***</td>
</tr>
</tbody>
</table>

Log-likelihood
Observations: 412747, 383029
$\rho$: -0.22, -1.00
$\sigma_e$: 11.60, 50.81
$\beta_3$: -2.59, -50.81

*** significant at 1% level; ** significant at 5% level; * significant at 10% level.
Note: node $i$ refers to the node based on which the observation unit is identified.
### Progression

<table>
<thead>
<tr>
<th>Time Group</th>
<th>Delay</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground buffer in time group 1</td>
<td>-0.006**</td>
<td>-0.047***</td>
</tr>
<tr>
<td>Ground buffer in time group 2</td>
<td>-0.013***</td>
<td>-0.061***</td>
</tr>
<tr>
<td>Ground buffer in time group 3</td>
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<td>Flight buffer in time group 1</td>
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<td>Delay_21</td>
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<td>Constant</td>
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<td>p-value</td>
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**Note:** node $i$ refers to the node based on which the observation unit is identified.

*** significant at 1% level; ** significant at 5% level; * significant at 10% level.
Reference:


