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Choice-Based Airline Schedule Design and Fleet Assignment

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May 17th-19th, 2021

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Speaker Bio: Vikrant Vaze

- Vikrant Vaze is Stata Family Career Development Associate Professor and the Program Lead of Mechanical, Operations and Systems Engineering at Dartmouth College. He is also a Research Affiliate in the Department of Aeronautics and Astronautics at Massachusetts Institute of Technology (MIT).
- He received an MS in Transportation, another MS in Operations Research and a PhD in Transportation Systems, all from MIT. He has a BS in Civil Engineering from the Indian Institute of Technology Mumbai.
- He works on solving large-scale transportation problems including consumer choice modeling, schedule planning, and disruption management using optimization, game theory, and machine learning.
- He is the recipient the US National Science Foundation's Career Award, MIT Presidential Fellowship, President of India Gold Medal, Best PhD Dissertation Award from the Council of University Transportation Centers, and two best paper awards from the FAA/Eurocontrol Air Traffic Management Seminar.
- He and his advisees have won medals and honorable mentions in the AGIFORS Ana Valicek Best Paper Award Competition in years 2010, 2015, 2017 and 2019.

Introduction

- We study an integrated airline **fleet assignment** and **schedule design** with **passenger choice**.
  - Schedule Design: Which flights are selected to fly?

BOS → SFO, Departure: 10AM BOS → SFO, Departure: 8AM BOS → SFO, Departure: 2PM

  - Fleet Assignment: Which aircraft type is assigned to a flight?

BOS → SFO, Departure: 8AM

? ? ?

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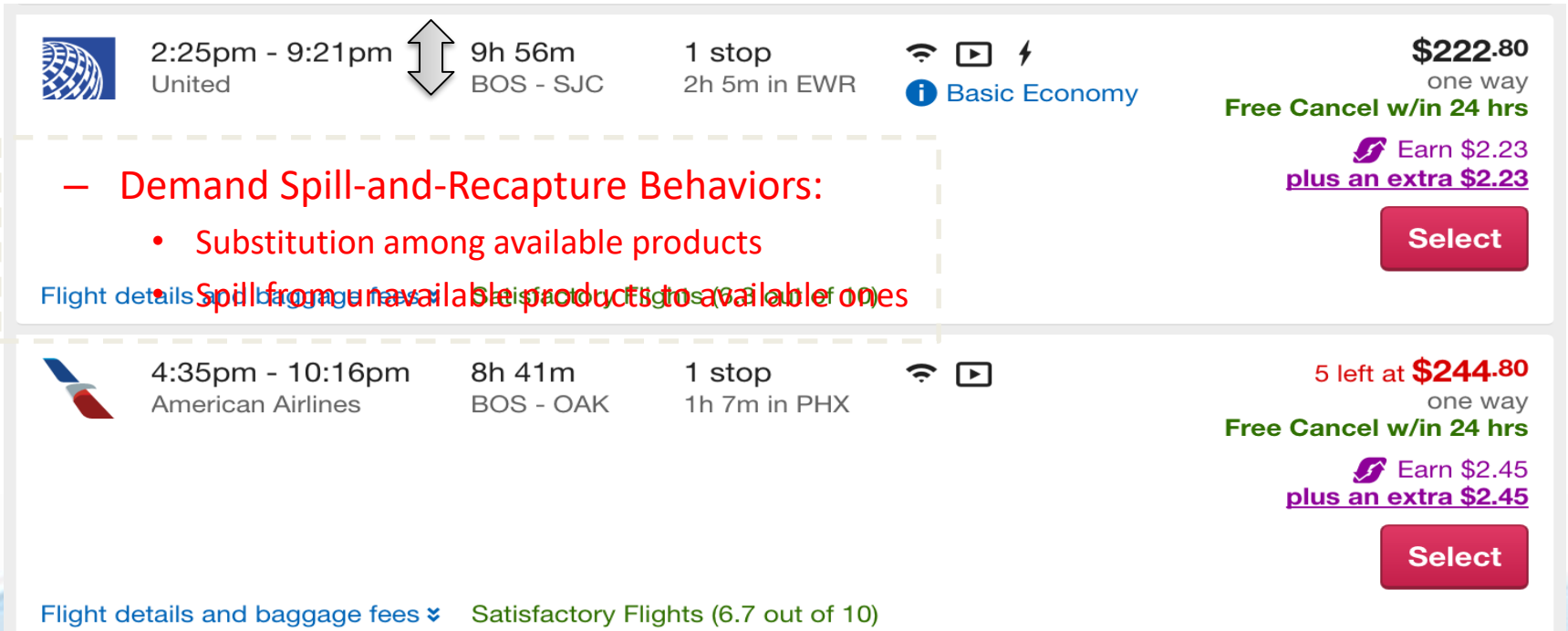
Introduction

- We study integrated airline **fleet assignment** and **schedule design**
  - Which flights are selected to fly?
  - Which aircraft type is assigned to a flight?

Heavily depends on the "Passenger Demand" for each flight.



- Demand Spill-and-Recapture Behaviors:
  - Substitution among available products
  - Spill from unavailable products to available ones

Introduction

- We study integrated airline **fleet assignment** and **schedule design** with **passenger choice**.

– Which flights are selected to fly?
– Which aircraft type is assigned to a flight?

Airline Network Planning

⇕

– Demand Spill-and-Recapture Behaviors:
  - Substitution among available products
  - Spill from unavailable products to available ones

Passenger Choice Modeling

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Demand Modeling – Multinomial Logit Model (MNL)

O-D Market

BOS

SFO

Fare Products	Attractiveness	Purchase Prob.
BOS-CLT-SFO Dep: 5:00 AM Arr: 12:12 PM Refundable, $930	1	5%
BOS-CLT-SFO Dep: 5:00 AM Arr: 12:12 PM Non-refundable, $451	3	15%
BOS-PHL-SFO Dep: 5:00 AM Arr: 10:47 AM Refundable, $1031	2	10%
BOS-PHL-SFO Dep: 5:00 AM Arr: 10:47 AM Non-refundable, $396	4	20%
Other airlines' products	10	50%

Demand Modeling – Multinomial Logit Model (MNL)

O-D Market

BOS

SFO

Fare Products	Attractiveness	Purchase Prob.
BOS-CLT-SFO Dep: 5:00 AM Arr: 12:12 PM Refundable, $930	1	6.67%
~~BOS-CLT-SFO Dep: 5:00 AM Arr: 12:12 PM Non-refundable, $451~~	3	0%
~~BOS-PHL-SFO Dep: 5:00 AM Arr: 10:47 AM Refundable, $1031~~	2	0%
BOS-PHL-SFO Dep: 5:00 AM Arr: 10:47 AM Non-refundable, $396	4	26.67%
Other airlines' products	10	66.67%

Problem Definition

- Given:
  - Flight schedule: a set of (optional) flight legs
  - Aircraft fleet: consisting of different fleet types
  - Passenger demand patterns
  - Fares and operating cost data
- Find:
  - A selection of flights as well as a feasible fleet assignment, that maximizes:

Profit = Total revenue – Total operating cost

Recent Work

- Wei, K., V. Vaze, and A. Jacquillat (2020). "Airline Timetable Development and Fleet Assignment Incorporating Passenger Choice". *Transportation Science*, 54(1), 139–163.
  - Timetable development from scratch ("clean slate")
  - Discrete choice model integrated into a large-scale optimization problem
  - Development of MIP-based solution heuristics: employing Decomposition, Variable Fixing, Symmetry Induction
- Ignores Revenue Management
  - Successfully handles instances with 1 fare product per itinerary
  - Not tractable under multiple products and multiple customer segments

Provides motivation for this talk

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A General Formulation for Choice-based Schedule Design and Fleet Assignment (CSD-FAM)

- Decision variable:
  - $x_{l,f} = 1$ if flight leg l is assigned with fleet type f; 0, otherwise

Choice-based revenue management

(CSD-FAM) $$V_{\text{CSD-FAM}} = \max \quad \boxed{r(\mathbf{x};\mathbf{h})} - \sum_{l\in\mathcal{L}} \sum_{f\in\mathcal{F}(l)} c_{l,f} x_{l,f}$$

Maximize: *profit=revenue-cost*

$$\text{s.t.} \quad \sum_{f\in\mathcal{F}(l)} x_{l,f} = 1, \quad \forall l \in \mathcal{L}_m,$$

Mandatory flights should be operated by one fleet type

$$\sum_{f\in\mathcal{F}(l)} x_{l,f} \le 1, \quad \forall l \in \mathcal{L}_o,$$

Optional flights can be cancelled

$$y_{v^-} + \sum_{l\in I(v)} x_{l,f} - y_v - \sum_{l\in O(v)} x_{l,f} = 0, \quad \forall f \in \mathcal{F}, \ \forall v \in N_f,$$

Aircraft flow balance

$$\sum_{v\in T_N(f)} y_v + \sum_{l\in T_F(f)} x_{l,f} \le n_f, \quad \forall f \in \mathcal{F},$$

Aircraft availability

$$y_v \ge 0, \quad \forall f \in \mathcal{F}, \ \forall v \in N_f, \tag{6}$$

$$x_{l,f} \in \{0,1\}, \quad \forall f \in \mathcal{F}(l), \ \forall l \in \mathcal{L}. \tag{7}$$

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Optimal Revenue for a Fixed Fleet Assignment

- Model demand substitutions via a **MNL choice model.**
  - h_p, price of fare product p.
  - v_p, utility of fare product p.
  - v_m, utility of competing airlines' products in O-D market m.
  - s_p, sale of fare product p.
  - s_m, sale of competing airlines' products in O-D market m.

$$
\begin{aligned}
\text{(SBLP)} \quad r(\mathbf{x};\mathbf{h}) = \max_{\mathbf{s}} \quad & \sum_{p\in\mathcal{P}} h_p s_p \\
\text{s.t.} \quad & \sum_{p\in\mathcal{P}} \delta_{l,p} s_p \le \sum_{f\in\mathcal{F}(l)} \mathrm{CAP}_f x_{l,f}, \ \forall l \in \mathcal{L}, \\
& \sum_{p\in\mathcal{P}(m)} s_p + s_m = \Lambda_m, \ \forall m \in \mathcal{M}, \\
& \frac{s_p}{v_p} - \frac{s_m}{v_m} \le 0, \ \forall m \in \mathcal{M}, \ \forall p \in \mathcal{P}(m), \\
& s_m, s_p \ge 0, \quad \forall m \in \mathcal{M}, \ \forall p \in \mathcal{P}. \qquad (12)
\end{aligned}
$$

Maximize: ***revenue***

Total sale on each flight <= capacity

Total sale of each O-D market = demand

Sale ratio / utility ratio consistency

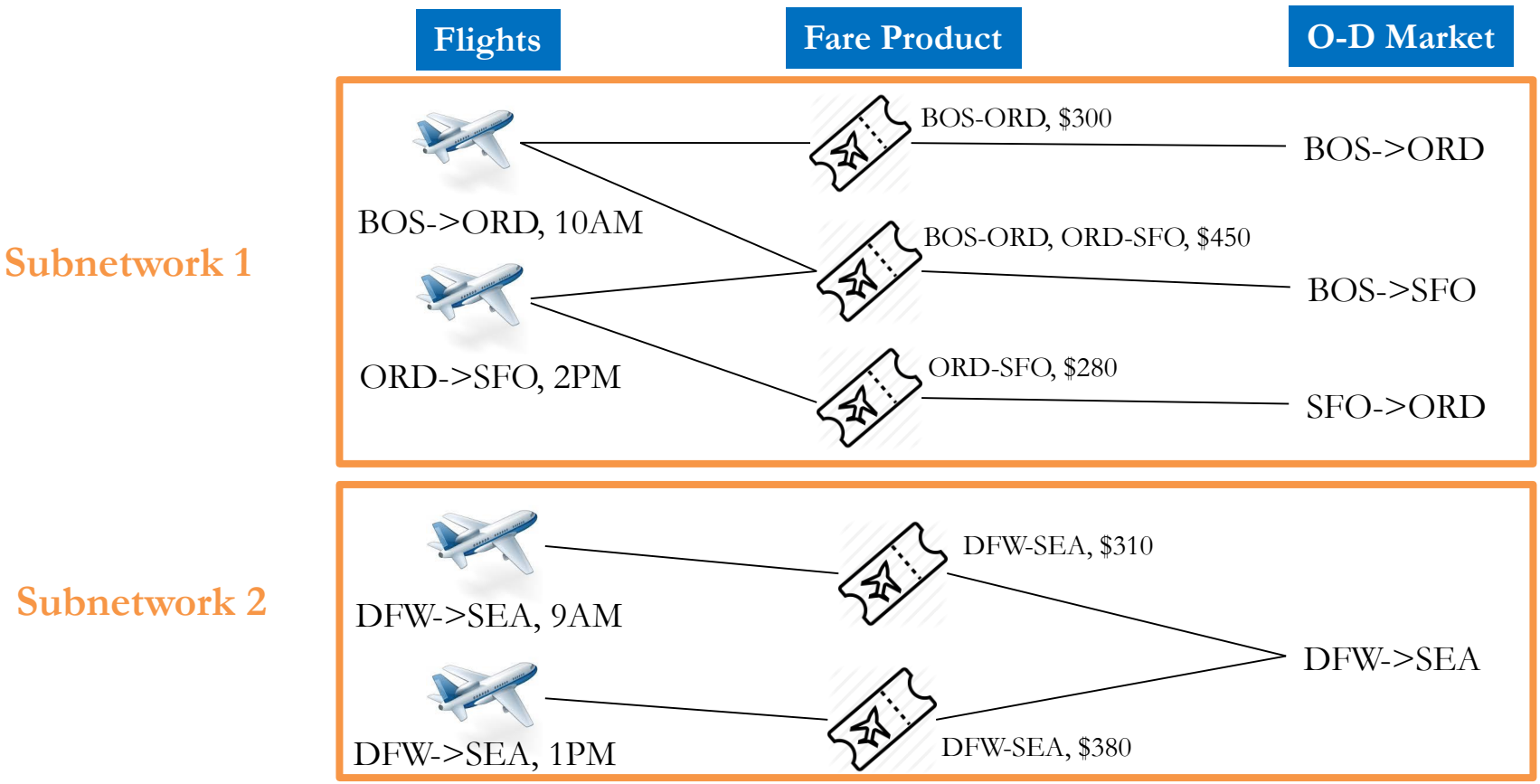
(Gallego, Ratliff and Shebalov 2014)

On CSD-FAM

- CSD-FAM is **notoriously** difficult to solve.
  - The difficulty comes from strong network effects.
  - Under the same computational budget, it often cannot produce a better solution compared to the one from a model which does not capture demand substitutions.

- **Goal:** develop a tractable approximation and decomposition scheme to solve CSD-FAM more effectively and efficiently.

Decomposition of the Problem



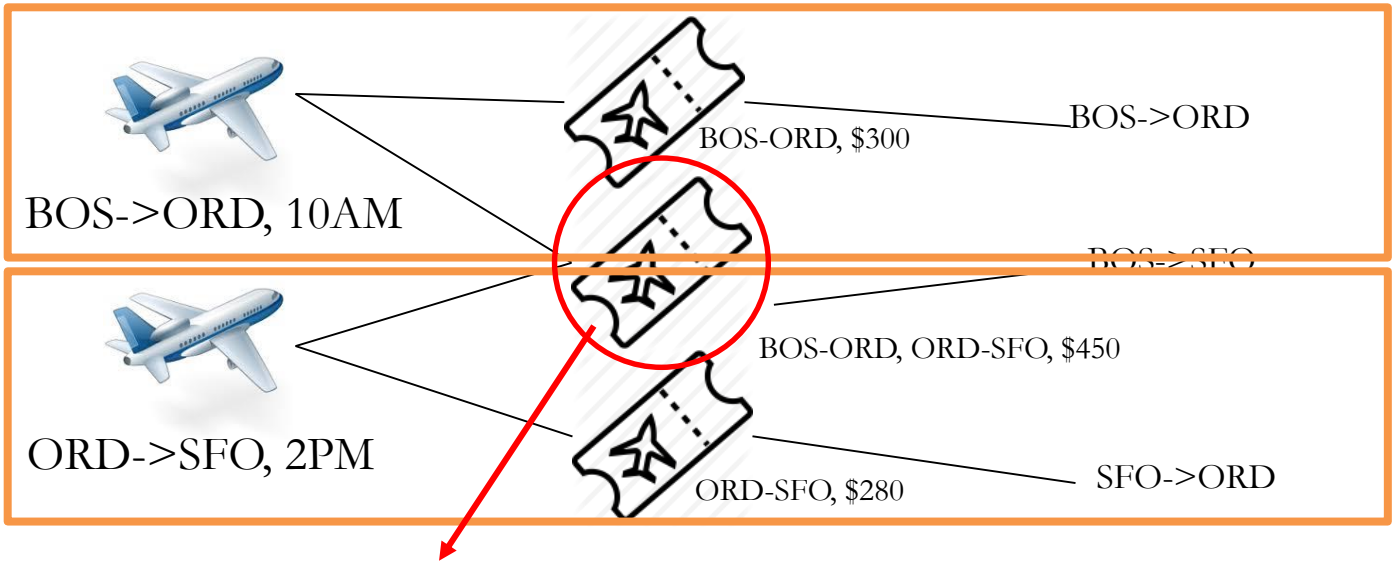
Proposition 1: With independent partition, subnetwork revenue does not depend on fleet assignment to flights not in the subnetwork, and total revenue can be perfectly decomposed as the sum of revenues of individual subnetworks.

Challenge

Such independent partition might give you very large sized subnetworks … ☹️

Approximation of the Problem

- How to decompose into smaller subnetworks without sacrificing optimality too much?



1. How to create subnetworks?

2. How to split the fare?

3. How to solve the decomposed CSD-FAM?

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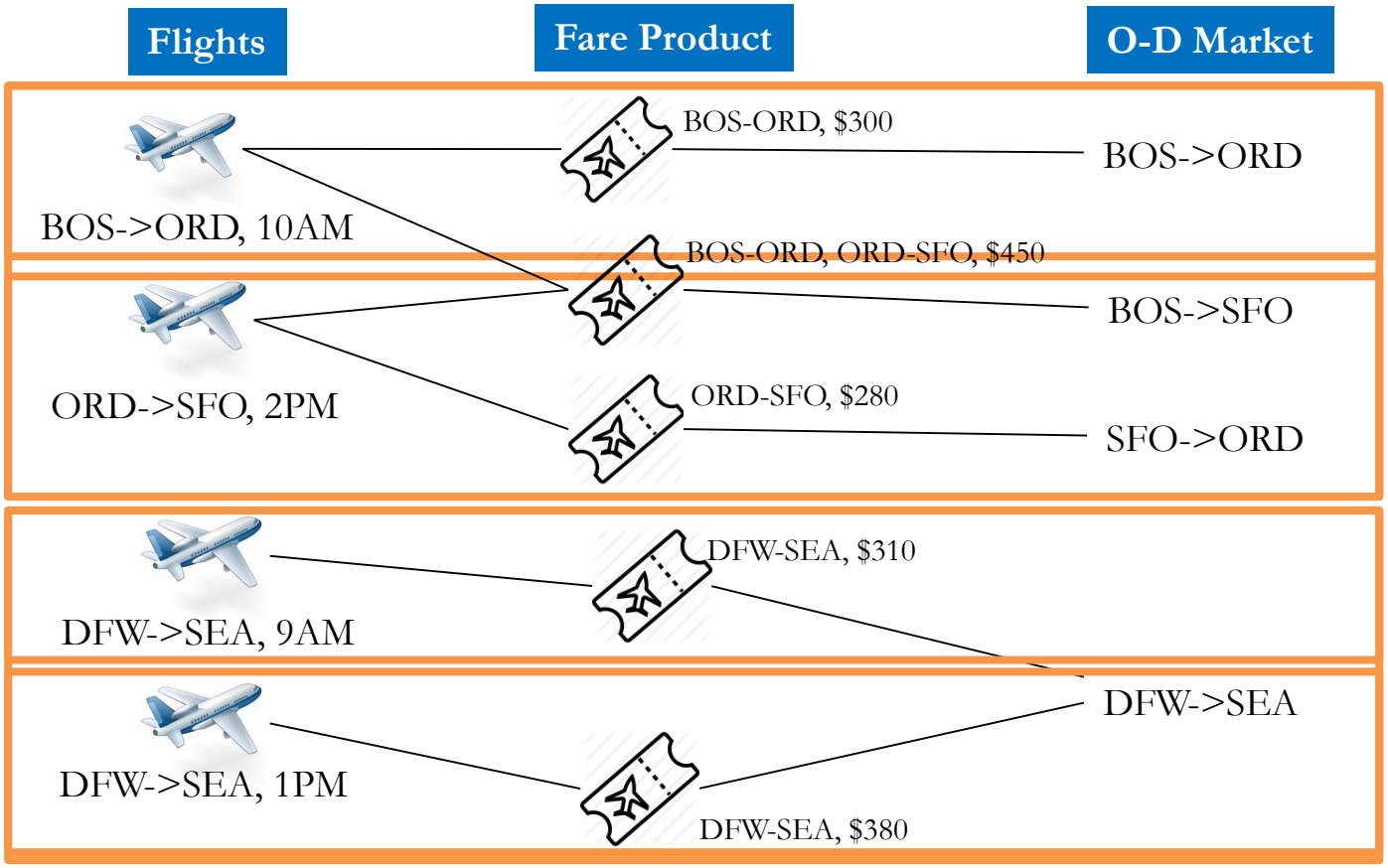
1. How to Create Subnetworks?



1. How to Create Subnetworks? (cont'd)

- **Key Result:**
  - Suppose the subnetworks are created from the maximally connected components after removing all products in some set $\hat{P}$,

  we have:

$$0 \leq \sum_{k=1}^{K} r^k\left(\boldsymbol{x}^{\boldsymbol{k}}, \boldsymbol{h}^{\boldsymbol{k}}\right) - r(\boldsymbol{x}, \boldsymbol{h}) \leq \boxed{\sum_{p \in \hat{P}} h_p \bar{\mu}_p}$$

Does not depend on the fare split.

  for any fleet assignment x, where $\bar{\mu}_p$ is the maximum sale of product p.

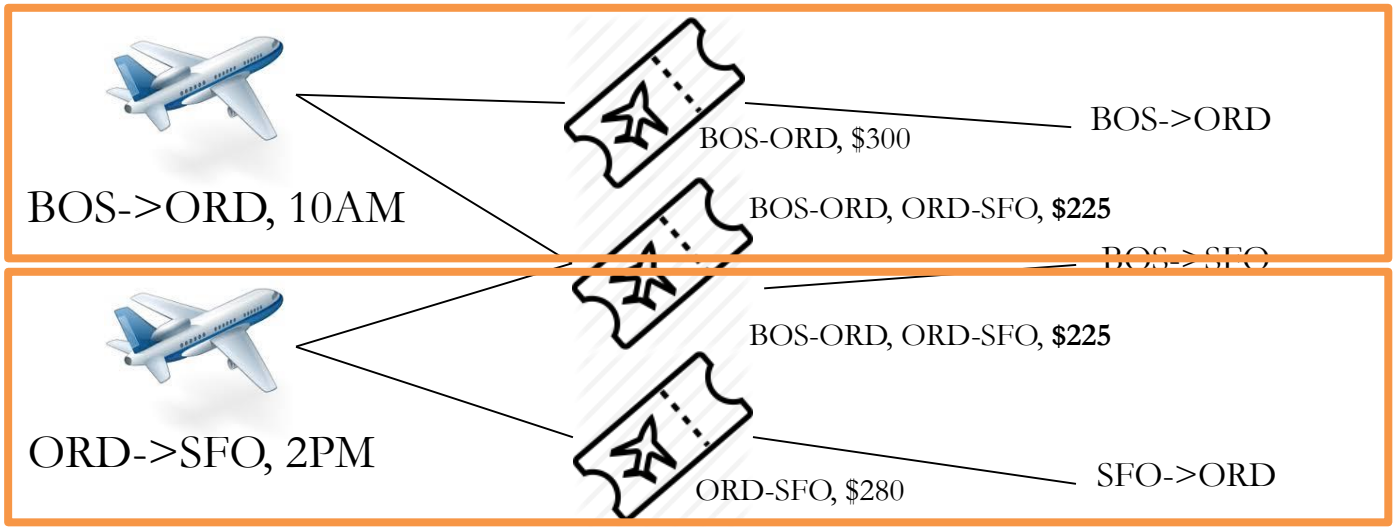
- **Heuristic Sketch:**
  - Removing product nodes in ascending order of their revenue contribution until certain level of fragmentation is reached.

2. How to Split the Fare?

- For each subnetwork i, define its price vector as h^i.

$$h = \begin{Bmatrix} 300 \\ 450 \\ 280 \end{Bmatrix} \rightarrow h^1 + h^2 = \begin{Bmatrix} 300 \\ 225 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 225 \\ 280 \end{Bmatrix}$$

Example:

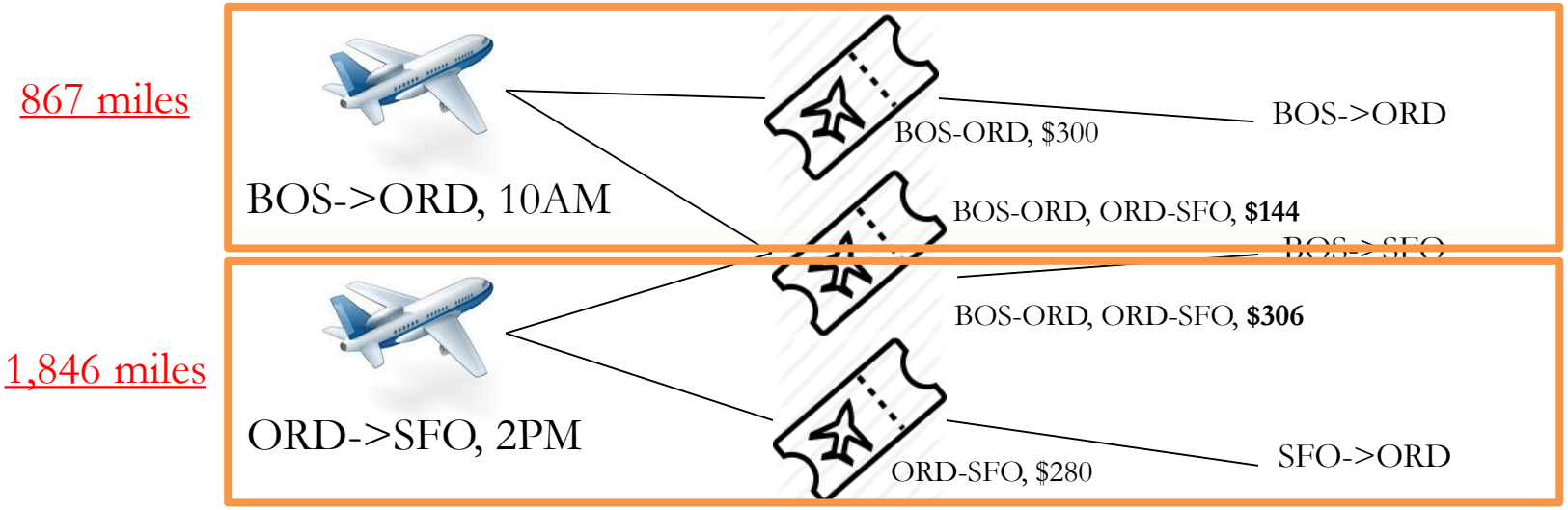


2. How to Split the Fare? (cont'd)

- Common heuristics: **distance-based proration.**

$$h = \left\{ \begin{array}{c} 300 \\ 450 \\ 280 \end{array} \right\} \rightarrow h^1 + h^2 = \left\{ \begin{array}{c} 300 \\ 144 \\ 0 \end{array} \right\} + \left\{ \begin{array}{c} 0 \\ 306 \\ 280 \end{array} \right\}$$

Example:



2. How to Split the Fare? (cont'd)

- **Back to our Key Result:**

$$0 \leq \sum_{k=1}^{K} r^k\left(\boldsymbol{x}^{\boldsymbol{k}}, \boldsymbol{h}^{\boldsymbol{k}}\right) - r(\boldsymbol{x}, \boldsymbol{h}) \leq \boxed{\sum_{p \in \hat{P}} h_p \bar{\pi}_p}$$

=> $$\boxed{\sum_{k=1}^{K} r^k\left(\boldsymbol{x}^{\boldsymbol{k}}, \boldsymbol{h}^{\boldsymbol{k}}\right)} \geq r(\boldsymbol{x}, \boldsymbol{h})$$

1. Choose subnetworks that minimize revenue bound
2. Find the optimal fare split to achieve the tightest upper bound:

$$\boxed{\min_{\boldsymbol{h}^{\boldsymbol{1}}, \ldots, \boldsymbol{h}^{\boldsymbol{K}}: \sum_k \boldsymbol{h}^{\boldsymbol{k}} = \boldsymbol{h}} \sum_{k=1}^{K} r^k\left(\boldsymbol{x}^{\boldsymbol{k}}, \boldsymbol{h}^{\boldsymbol{k}}\right)}$$

– **YES! This can be cast as a single linear program.**

2. How to Split the Fare? (cont'd)

- The fare split optimization problem is:

$$\min_{\boldsymbol{h}^1,\ldots,\boldsymbol{h}^K:\sum_k \boldsymbol{h}^k=\boldsymbol{h}} \sum_{k=1}^{K} r^k\left(\boldsymbol{x}^k, \boldsymbol{h}^k\right)$$

- **Taking the dual** of the inner maximization problem, this fare and market split optimization problem can be **reformulated as a linear program**.

- Under what fleet assignment $\boldsymbol{x}$ shall we optimize for the fare split?

2. How to Split the Fare? (cont'd)

- We solve:

$$\min_{\boldsymbol{h}^1,\ldots,\boldsymbol{h}^K:\sum_k \boldsymbol{h}^k=\boldsymbol{h}} \max_{x\in\chi} \sum_{k=1}^{K} r^k\left(\boldsymbol{x}^k,\boldsymbol{h}^k\right)$$

where χ is the set of all feasible fleet assignments.

- **This is a very hard problem to solve.**
- **But with linear relaxation of the inner maximization can be dualized to efficiently provide a potentially looser but still valid revenue upper bound.**

$$(\widetilde{\mathrm{FS}}) \quad \tilde{V}_{\mathrm{FS}}(\Pi) = \min_{\sum_{k=1}^{K} \mathbf{h}^k=\mathbf{h}} \tilde{V}_{\text{CSD-FAM}(\Pi,\mathcal{H})}. \tag{33}$$

3. How to Solve the Decomposed CSD-FAM?

- Revenue function is decomposed into the sum of subnetwork revenue based on the local fare vector h^i.

$$\max \quad r(\mathbf{x};\mathbf{h}) - \sum_{l\in\mathcal{L}} \sum_{f\in\mathcal{F}(l)} c_{l,f} x_{l,f}$$

- We develop further reformulation technique to tighten its LP relaxation bound based on a Dantzig-Wolfe type reformulation.

$$\max \quad \sum_{k=1}^{K} r^k(\mathbf{x}^k;\mathbf{h}^k) - \sum_{l\in\mathcal{L}} \sum_{f\in\mathcal{F}(l)} c_{l,f} x_{l,f}$$

Moreover, we can avoid column generation, by using a variable reduction technique based on parsimonious enumeration (Barnhart et al., 2009).

Subnetwork-based CSD-FAM (S-CSD-FAM)

- Utilize composite variable to isolate network effect (Barnhart et al. 2009).
  - Two actual fleet types: A, B.
  - $\emptyset \cong$ not assign any fleet type

Subnetwork 1	Fleet Assignment								
Flight	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
BOS->ORD	A	B	$\emptyset$	A	B	$\emptyset$	A	B	$\emptyset$
ORD->SFO	A	A	A	B	B	B	$\emptyset$	$\emptyset$	$\emptyset$

  - Define: $w_j = 1$ if assignment j is applied.

- Better formulation with tightened LP relaxation.
- We find that a mixed approach works even better:
  - Mixed-subnetwork based CSD-FAM
  - Decompose only a part of the network

Decomposed Formulation

- Revenue function is decomposed into the sum of subnetwork revenue based on the local fare vector h^i.

(CSD-FAM)

$$V_{\text{CSD-FAM}} = \max \quad r(\mathbf{x};\mathbf{h}) - \sum_{l\in\mathcal{L}} \sum_{f\in\mathcal{F}(l)} c_{l,f} x_{l,f}$$

(CSD-FAM(Π,$\mathcal{H}$))

$$V_{\text{CSD-FAM}(\Pi,\mathcal{H})} = \max \quad \sum_{k=1}^{K} r(\mathbf{x}^k;\mathbf{h}^k) - \sum_{l\in\mathcal{L}} \sum_{f\in\mathcal{F}(l)} c_{l,f} x_{l,f}$$

Flight-Fleet Based Representation

(S-CSD-FAM(Π, $\mathcal{H}$))

$$V_{\text{S-CSD-FAM}(\Pi,\mathcal{H})} = \max \quad \sum_{k=1}^{K} \sum_{j\in\mathcal{X}^k} (r_j - c_j) w_j$$

Composite Variable Representation

(S-CSD-FAM(Π_c, Π_t, $\mathcal{H}$))

Mixed Representation

$$V_{\text{S-CSD-FAM}(\Pi_c,\Pi_t,\mathcal{H})} = \max \quad \sum_{k:\mathcal{L}^k\in\Pi_c} \sum_{j\in\mathcal{X}^k} (r_j - c_j) w_j + \sum_{k:\mathcal{L}^k\in\Pi_t} r(\mathbf{x}^k;\mathbf{h}^k) - \sum_{k:\mathcal{L}^k\in\Pi_t} \sum_{l\in\mathcal{L}^k} \sum_{f\in\mathcal{F}(l)} c_{l,f} x_{l,f}$$

Theorem 1: Size-Strength Tradeoff

- As more subnetworks in the mixed formulation are represented by composite variables, strength increases but problem size also grows.

$$V_{\text{CSD-FAM}(\Pi,\mathcal{H})} = V_{\text{S-CSD-FAM}(\Pi_c,\Pi_t,\mathcal{H})} = V_{\text{S-CSD-FAM}(\Pi,\mathcal{H})},$$

LP Relaxation ↓ LP Relaxation ↓ LP Relaxation ↓

$$\tilde{V}_{\text{CSD-FAM}(\Pi,\mathcal{H})} \geq \tilde{V}_{\text{S-CSD-FAM}(\Pi_c,\Pi_t,\mathcal{H})} \geq \tilde{V}_{\text{S-CSD-FAM}(\Pi,\mathcal{H})}.$$

Moreover, for any two splits (Π_c, Π_t) *and* (Π'_c, Π'_t) *of the same partition* Π *with* $\Pi_c \supset \Pi'_c$ *and* $\Pi_t \subset \Pi'_t$,

$$\boxed{\tilde{V}_{\text{S-CSD-FAM}(\Pi'_c,\Pi'_t,\mathcal{H})} \geq \tilde{V}_{\text{S-CSD-FAM}(\Pi_c,\Pi_t,\mathcal{H})}.}$$

Overall Solution Algorithm

Solving S-CSD-FAM$(\Pi_c, \Pi_t, \mathcal{H})$

Step 1 Run a partition algorithm by specifying approximation quality to construct a partition $\{\mathcal{L}^1, \cdots, \mathcal{L}^K\}$ of subnetworks and assign each subnetwork to either Π_c or Π_t.

Step 2 Solve the linear programming formulation of the relaxed fare split problem $\widetilde{\mathsf{FS}}$ stated in (33) to get local fare vectors $\mathbf{h}^k, \forall k \in \{1, \cdots, K\}$.

Step 3 Trim the set of all possible local fleet assignments $\mathcal{X}^k$ using a variable reduction technique; calculate $r_j, \forall j \in \mathcal{X}^k, \forall k \in \{1, \cdots, K\} : \mathcal{L}^k \in \Pi_c$.

Step 4 Solve the S-CSD-FAM$(\Pi_c, \Pi_t, \mathcal{H})$ formulation

Computational Results

- Two instances:

Instance	Flights	Itineraries	Fare Products	Markets	Fleet Types	Aircraft	Market Share
D1	180	1,195	10,645	1,648	9	71	20.7%
D2	815	4,290	47,190	819	7	186	45.9%

- Evaluate profit of our solution S-CSD-FAM(Π_c, Π_t, H) against:
  - CSD-FAM: choice-based formulation without decomposition
  - ISD-FAM: independent-demand formulation without decomposition
  - ISD-FAM-SR-ITIN: ISD-FAM with spill and recapture
  - CSD-FAM(Π, H): choice-based formulation with decomposition but without composite variables
- Next slide reports results after 5 Hr CPU time
- All revenues reported in Million Dollars per Year

Computational Results: D1

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Inst.	Model	Fare Split	Obj. Val.	LP Relax.	CPU Time	Solver Gap	B&B Nodes	ΔProfit
D1	ISD-FAM	-	2.87	3.36	304	0.00%	2,186	0.00
	ISD-FAM-SR-ITIN	-	2.26	2.57	10,503	0.00%	194,785	(30.90)
	CSD-FAM	-	3.66	4.36	LIMIT	0.98%	131,202	3.74
	CSD-FAM($\Pi^0, \mathcal{H}$)	dist	3.75	4.44	LIMIT	0.90%	51,934	1.31
		opt	3.73	4.38	LIMIT	0.76%	59,566	5.22
	S-CSD-FAM($\Pi^0_c, \Pi^0_t, \mathcal{H}$)	dist	3.75	4.31	LIMIT	0.60%	72,898	1.31
		opt	3.73	4.28	LIMIT	0.44%	73,837	5.22
	CSD-FAM($\Pi^1, \mathcal{H}$)	dist	4.00	4.64	17,952	0.00%	90,044	(4.52)
		opt	3.82	4.40	17,292	0.00%	170,836	5.62
	S-CSD-FAM($\Pi^1_c, \Pi^1_t, \mathcal{H}$)	dist	4.00	4.44	1,837	0.00%	43,280	(4.52)
		opt	3.82	4.26	1,807	0.00%	39,768	**5.62**
	CSD-FAM($\Pi^2, \mathcal{H}$)	dist	4.17	4.83	8,111	0.00%	47,411	(30.33)
		opt	3.93	4.45	3,193	0.00%	82,840	1.39
	S-CSD-FAM($\Pi^2_c, \Pi^2_t, \mathcal{H}$)	dist	4.17	4.23	25	0.00%	0	(30.33)
		opt	3.93	3.97	25	0.00%	0	1.39

Computational Results: D2

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Inst.	Model	Fare Split	Obj. Val.	LP Relax.	CPU Time	Solver Gap	B&B Nodes	ΔProfit
D2	ISD-FAM	-	4.25	4.50	LIMIT	0.03%	27,727	0.00
	ISD-FAM-SR-ITIN	-	3.94	4.15	LIMIT	2.21%	7,992	(41.55)
	CSD-FAM	-	6.65	7.25	LIMIT	6.48%	7,109	11.98
	CSD-FAM($\Pi^0, \mathcal{H}$)	dist	6.76	7.31	LIMIT	5.85%	6,922	20.02
		opt	6.77	7.27	LIMIT	5.06%	7,160	35.41
	S-CSD-FAM($\Pi^0_c, \Pi^0_t, \mathcal{H}$)	dist	6.79	7.26	LIMIT	5.13%	7,022	25.51
		opt	6.72	7.22	LIMIT	5.68%	6,214	42.25
	CSD-FAM($\Pi^1, \mathcal{H}$)	dist	6.83	7.37	LIMIT	5.60%	6,683	11.68
		opt	6.85	7.30	LIMIT	4.13%	7,509	38.60
	S-CSD-FAM($\Pi^1_c, \Pi^1_t, \mathcal{H}$)	dist	6.90	7.31	LIMIT	3.67%	9,522	42.99
		opt	6.86	7.24	LIMIT	3.96%	6,634	**47.15**
	CSD-FAM($\Pi^2, \mathcal{H}$)	dist	6.88	7.47	LIMIT	6.32%	7,209	(11.52)
		opt	6.86	7.35	LIMIT	5.20%	7,504	14.04
	S-CSD-FAM($\Pi^2_c, \Pi^2_t, \mathcal{H}$)	dist	7.01	7.39	LIMIT	3.56%	7,432	26.68
		opt	6.96	7.28	LIMIT	2.79%	10,046	45.52

Under Booking Simulation and Imperfect Demand Forecasts

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Inst.	Perturb.	Scenario	ISD-FAM-SR-ITIN		CSD-FAM		S-CSD-FAM($\Pi_c^1, \Pi_t^1, \mathcal{H}$)	
			ΔProfit(SBLP)	ΔProfit(Sim)	ΔProfit(SBLP)	ΔProfit(Sim)	ΔProfit(SBLP)	ΔProfit(Sim)
D1	±0%	1	(30.90)	(25.75) ± 6.22	3.74	5.65 ± 6.18	5.62	**5.93 ± 6.16**
	±10%	1	(29.92)	(27.44) ± 6.79	3.43	**6.47 ± 6.45**	6.65	5.72 ± 6.47
		2	(31.08)	(27.44) ± 6.51	3.60	5.19 ± 6.95	4.60	**6.62 ± 6.96**
		3	(30.76)	(24.69) ± 6.13	3.03	0.54 ± 6.59	5.34	**5.23 ± 6.04**
		4	(29.90)	(26.97) ± 6.20	2.66	2.16 ± 6.24	5.82	**7.11 ± 6.22**
		5	(31.13)	(29.32) ± 6.60	2.65	0.59 ± 6.30	4.34	**5.01 ± 6.23**
	±25%	1	(28.14)	(23.49) ± 6.61	3.02	4.64 ± 6.97	8.19	**4.80 ± 6.66**
		2	(31.16)	(30.45) ± 5.97	3.26	**6.30 ± 5.91**	2.95	4.00 ± 5.90
		3	(30.86)	(24.18) ± 7.06	1.24	2.42 ± 6.77	4.25	**6.52 ± 6.68**
		4	(27.40)	(26.29) ± 7.53	0.99	(0.20) ± 7.14	6.69	**5.53 ± 7.03**
		5	(30.74)	(29.11) ± 6.46	1.07	(0.48) ± 6.16	3.01	**6.16 ± 6.11**
	±50%	1	(25.18)	(21.28) ± 6.78	2.86	2.84 ± 7.28	3.60	**3.70 ± 6.80**
		2	(30.08)	(27.67) ± 7.46	2.54	**5.01 ± 6.58**	0.05	(3.12) ± 6.81
		3	(29.68)	(28.15) ± 6.28	(1.86)	(2.76) ± 6.05	2.56	**0.41 ± 6.51**
		4	(22.19)	(19.39) ± 7.03	(1.63)	(6.14) ± 7.21	8.59	**2.57 ± 6.71**
		5	(29.50)	(26.77) ± 6.59	(1.24)	(2.01) ± 6.61	0.85	**0.34 ± 6.49**
D2	±0%	1	(41.55)	(41.42) ± 4.48	11.98	3.37 ± 4.63	47.15	**40.11 ± 4.64**
	±10%	1	(41.86)	(42.54) ± 4.83	10.87	0.28 ± 4.28	46.67	**38.35 ± 4.67**
		2	(41.64)	(42.89) ± 5.01	9.21	2.46 ± 4.65	44.96	**37.10 ± 5.11**
		3	(40.74)	(39.57) ± 4.48	11.95	2.35 ± 4.37	46.78	**36.83 ± 4.41**
		4	(40.53)	(39.33) ± 5.30	14.25	6.57 ± 4.73	49.85	**41.71 ± 4.71**
		5	(41.13)	(43.51) ± 4.69	11.35	4.49 ± 4.42	46.96	**36.26 ± 4.86**
	±25%	1	(42.38)	(38.89) ± 4.53	8.61	5.22 ± 4.47	43.29	**36.82 ± 4.64**
		2	(42.19)	(41.64) ± 4.74	3.81	(5.31) ± 4.37	40.52	**30.70 ± 4.81**
		3	(39.51)	(35.40) ± 4.64	11.26	6.05 ± 4.09	44.83	**36.51 ± 4.54**
		4	(38.22)	(41.91) ± 4.56	16.84	5.13 ± 4.36	51.57	**38.38 ± 4.63**
		5	(40.54)	(40.67) ± 4.70	8.60	3.14 ± 4.71	45.52	**39.64 ± 4.67**
	±50%	1	(41.45)	(42.16) ± 4.44	4.57	(0.30) ± 4.47	43.15	**35.68 ± 4.64**
		2	(44.00)	(41.86) ± 4.67	(7.38)	(11.45) ± 4.56	28.08	**24.38 ± 4.28**
		3	(36.06)	(37.97) ± 4.62	6.57	(2.31) ± 4.17	39.04	**27.10 ± 4.94**
		4	(34.68)	(36.15) ± 4.47	13.88	5.35 ± 4.44	46.45	**37.15 ± 4.41**
		5	(37.18)	(36.32) ± 4.37	2.76	(0.80) ± 4.46	41.71	**34.45 ± 4.38**

Summary

- We provide a tractable decomposition of the choice-based integrated airline fleet assignment and schedule design.
- Demonstrate significant, consistent and robust profit improvements over all existing methods.
- Choice-based airline network planning represents a promising and fruitful future direction.
- Possible implementation at industry partner.

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Thank You!
Questions?